THE ANALYSIS OF WEDGED FRAMES
WITH BENT MEMBERS BY THE
STRING POLYGON METHOD

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THE ANALYSIS OF WEDGED FRAMES
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PREFACE

The material presented in this thesis is the outgrowth of seminar lectures delivered by Professor Jan J. Tuma in the springs of 1959 and 1960. The literature and the general theory of the string polygon for straight and bent members were presented by Professor Jan J. Tuma.

The application of the String Polygon Method to the analysis of wedged frame with bent members of variable cross section is presented in this thesis.

I wish to express sincere indebtedness and appreciation to the following:

To Professor Jan J. Tuma for his invaluable assistance and guidance in the preparation of this thesis and for acting as the writer's major advisor.

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To Mr. Paul E. Pulley, Staff Assistance of Computer center, for computing the tables of beam constants in the fourth chapter of this thesis.

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NOMENCLATURE

a_1, a_2, a_3. . . . . Dimensionless Terms

b_1, b_2, b_3. . . . . Dimensionless Terms

b . . . . Width of Beam

c_{11}, c_{12}, c_{13} . . . . Coefficients of Force Matrix

d_A. . . . . Minimum Depth of Beam

δd_A . . . . Maximum Depth of Beam

d_j, d_k . . . . Length of members <u>ij</u>, <u>jk</u>.

d_x. . . . . Depth of Beam at x

f . . . . Angular Flexibility Coefficient

g . . . . Angular Carry-Over Coefficient

i, j, k . . . . Letters Designating Joint or Intermediate Supports

n . . . . Moving Load Position Coefficient

q . . . . Maximum Intensity of Triangular Load

t_1, t_2, t_3. . . . . Angular Load Function Coefficients

t_f. . . . . Thickness of Flange

t_w. . . . . Thickness of Web

$u, v, \bar{u}, \bar{v}$,. . . . . Coordinates of Cross Section

$x, y, \bar{x}, \bar{y}, \bar{x}_{kj}, \bar{y}_{kj}$ . . . . Coordinates of Cross Section

A, B, C . . . . Letters Designating Supports of Frame

BM_x, BM_j . . . . Bending Moments at x, j Due to Loads

E . . . . .Modulus of Elasticity

F_{ij}, F_{ji} . . . . .Angular Flexibilities

G_{ij}, G_{ji} . . . . .Angular Carry-Over Values

I_o^* . . . . .Minimum Moment of Inertia of Beam

I_o . . . . .Minimum Moment of Inertia of Structure

I_x, I_u, I_t. . . . . .Moment of Inertias at x,u,t.

L_j, L_k . . . . .Length of Spans <u>ij</u>,<u>jk</u>.

(LL) . . . . .Due to Live Load (Unit Moving Load)

M_i, M_j, M_k. . . . . .Bending Moments at i,j,k,

$\overline{M}_x, \overline{M}_y, \overline{M}_{\bar{x}_k}, \overline{M}_{\bar{y}_k}$ . . . . .Moments of Elastic Weights with Respect to $x, y, \bar{x}_k, \bar{y}_k$.

$\overline{P}$ . . . . .Elastic Weight

P . . . . .Intensity of Moving Load

Q . . . . .Integration Constant

$\overline{R}$ . . . . .Elastic Reaction

(TL) . . . . .Due to Triangular Load

(UL) . . . . .Due to Uniformly Distributed Load

$\overline{W}_j$. . . . . .Total Elastic Weight of Segment <u>ij</u>

ρ . . . . .Shape Factor

ψ, δ . . . . .Dimensionless Ratios

ϕ_{ij} . . . . .Angle Between String Line <u>ij</u> and Tangent to Elastic Curve at j

ϕ_j . . . . .Change in Angle of String Line at j

θ_A, θ_B . . . . .End Slope at A or B

τ_o . . . . .Angular Load Function

Δ . . . . .Relative Displacement

Σ . . . . .Summation

μ . . . . Dimensionless Ratio

CHAPTER I

STRING POLYGON METHOD

1-1. Development of Method

The String Polygon Method for the analysis of various structural systems was developed by Jan J. Tuma and his students at the Oklahoma State University. The first formulation of the joint elastic weights as a basic function of the deformation string polygon was presented by J.J. Tuma (1) in his seminar CE620 in Spring of 1959. The application was limited to simple, compound and continuous beams. S.L. Chu, developed by means of the string polygon, formulas for angular beam functions (2) in the summer of 1959. In spring of 1960, J.J. Tuma (3) extended the theory of the string polygon to polygonal bars, rings and frames and suggested applications. A.F. Madayag, applied the string polygon method to the analysis of tapered airplane wings (4). J.T. Oden, and H.C. Boeker summarized the principles of the string polygon as applied to single span frames (5,6) and Oden recorded the application of the elastic center in connection with this method (5). J.W. Harvey, reported the analysis of column-beams by the same method (7). J.J. Tuma presented the extension of this approach to multi-panel frames in his lectures to the participants of NSF Summer Institute for College teachers of Civil Engineering in the summer of 1960 (8) and in a joint paper with Oden (9). Additional contributions were made by Exline (10), Gonulson (11) and Houser (12). F.N. Gauger, extended the string polygon to the plastic range and coined new terminology for this range (13). The generalization of the string polygon method for straight and bent members in space was presented by Tuma (14).

1-2. Geometry of String Polygon

The basic principles of the string polygon are fully discussed elsewhere (1). Some of these ideas must be restated here for the sake of clear presentation.

(a) Elemental Elastic Load $d\overline{W}$ is the change in slope of the element of an elastic member due to bending moment Ms (Figure 1-1).

$$d\,\phi_u = \frac{Mudu}{EI_u} = \overline{dWu} \qquad (1\text{-}1)$$

Where

E = Modulus of Elasticity

Iu = Moment of Inertia

The graphical representation of this angle is a force-vector acting perpendicular to the plane of the bar.

(b) Segmental Elastic Load $\overline{W}j$ is the change in slope of the segment d_j of an elastic member due to bending defined by the bending moment diagram (Figure 1-2).

$$\int_i^j d\,\phi_u = \int_i^j \frac{Mudu}{EI_u} = \overline{W}_j \qquad (1\text{-}2)$$

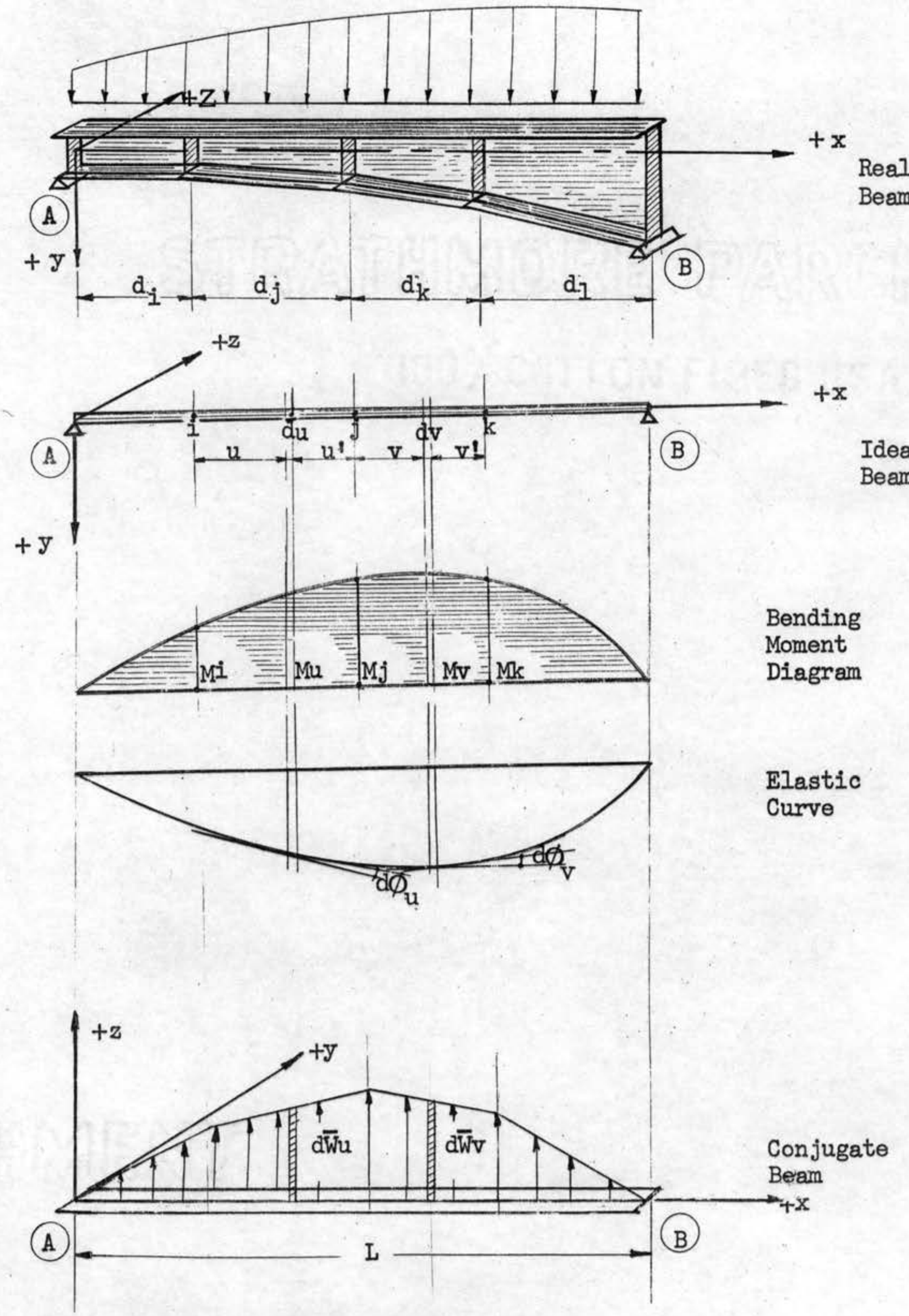


Fig. 1-1. Elemental Elastic Load

The graphical representation of this angle change is an area-vector acting perpendicular to the plane of the bar or a resultant force-vector of the same direction.

(c) Segmental Elastic Reactions $\bar{\bar{R}}$'s are the end slopes of the segment described above (Figure 1-2).

$$\phi_{ij} = \int_i^j \frac{M_u u' du}{d_j EI_u} = \bar{\bar{R}}_{ij}$$

$$\phi_{ji} = \int_i^j \frac{M_u u du}{d_j EI_u} = \bar{\bar{R}}_{ji} \qquad (1\text{-}3)$$

The graphical representation of these end slopes are force-vectors acting perpendicular to the bar.

(d) Joint Elastic Weight $\bar{\bar{P}}$ is the change in slope of the string lines of two adjacent segments of the bar (Figure 1-3).

$$\phi_j = \phi_{ji} + \phi_{jk} = \bar{\bar{P}}_{ji} + \bar{\bar{P}}_{jk} = \bar{\bar{P}}_j \qquad (1\text{-}4)$$

The graphical representation of this change in slope is the force-vector acting perpendicular to the plane of the bar.

These joint elastic weights represent a new set of force-vectors in a state of equilibrium and equivalent to the initial set of elemental elastic weights.

1-3. Elasto-Static Equilibrium

The joint elastic weights applied on the conjugate beam represent a new set of force-vectors in a state of static equilibrium and equivalent to the initial set of elemental elastic weights. Thus,

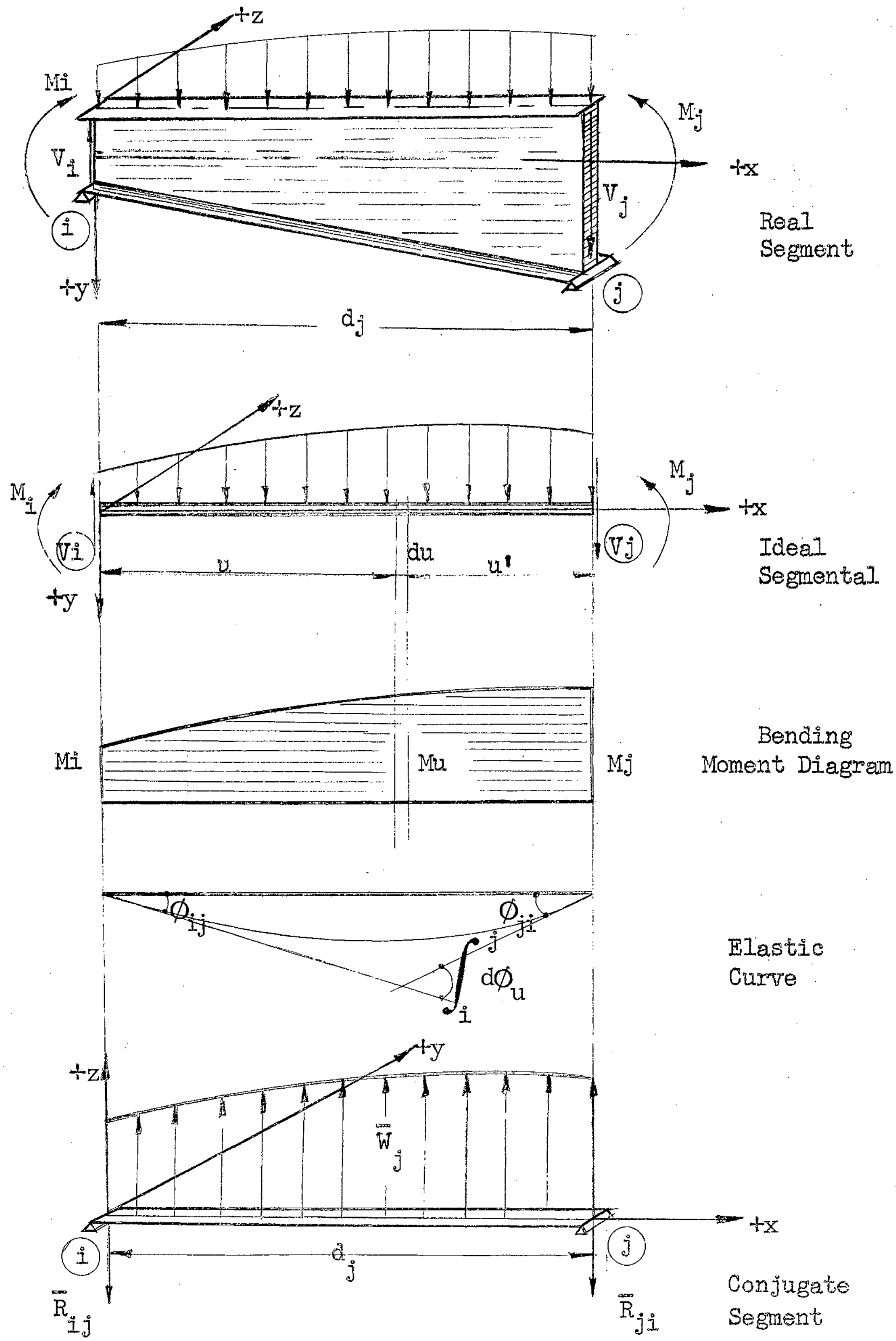


Figure 1-2. Segmental Elastic Load

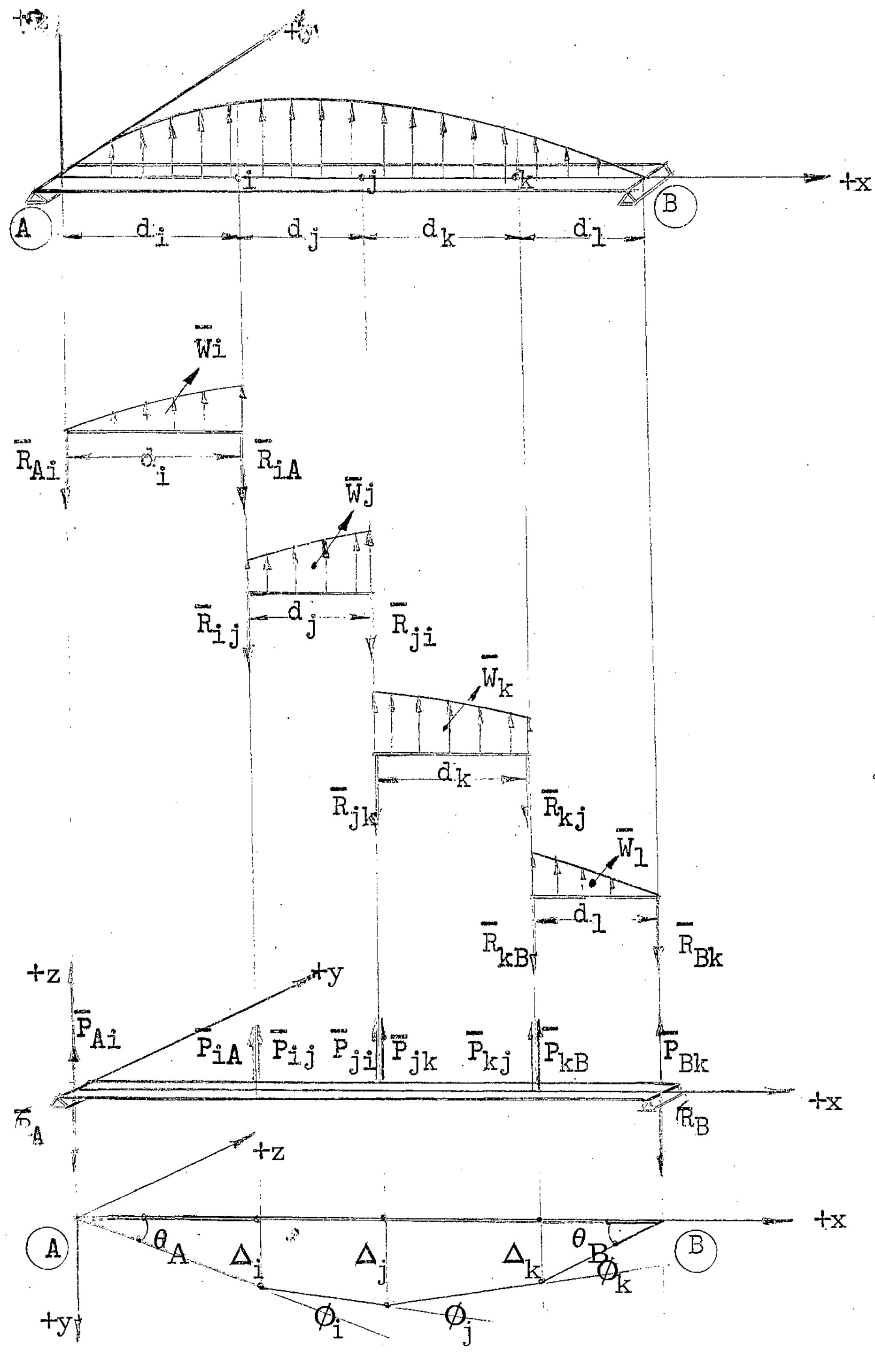


Figure 1-3. Joint Elastic Weights

$$\sum P_j = \sum \bar{R}'s \tag{1-5}$$

$$\sum \bar{M}_A = 0; \qquad L\bar{R}_B - \Sigma P_j x = 0 \tag{1-6}$$

$$\sum \bar{M}_B = 0; \qquad L\bar{R}_A - \Sigma P_j x' = 0 \tag{1-7}$$

1-4. Conjugate Frame Theorem

The algebric expressions for the joint elastic weights for a rigid frame (Figure 1-4), are developed from the elemental elastic weights (Figure 1-5) and segmental elastic weights (Figure 1-6) as shown in (Figure 1-7).

The operations with these elastic weights are described elsewhere (9).

The joint elastic weights applied on the conjugate frame represent a new set of force-vectors again necessarily in a state of static equilibrium and equivalent to the initial set of elemental elastic weights.

$$\begin{aligned} \sum \bar{P}_j &= 0 \\ \sum \bar{P}_j x &= 0 \\ \sum \bar{P}_j y &= 0 \end{aligned} \tag{1-8}$$

These formulas are very useful for calculation of bending moments and deformations.

1-5. Frame Joint Elastic Weight

The joint elastic weight given by Equation (1-4) as

$$\bar{P} = M_i G_{ij} + M_j \Sigma F_j + M_k G_{kj} + \Sigma \tau_j \tag{1-9a}$$

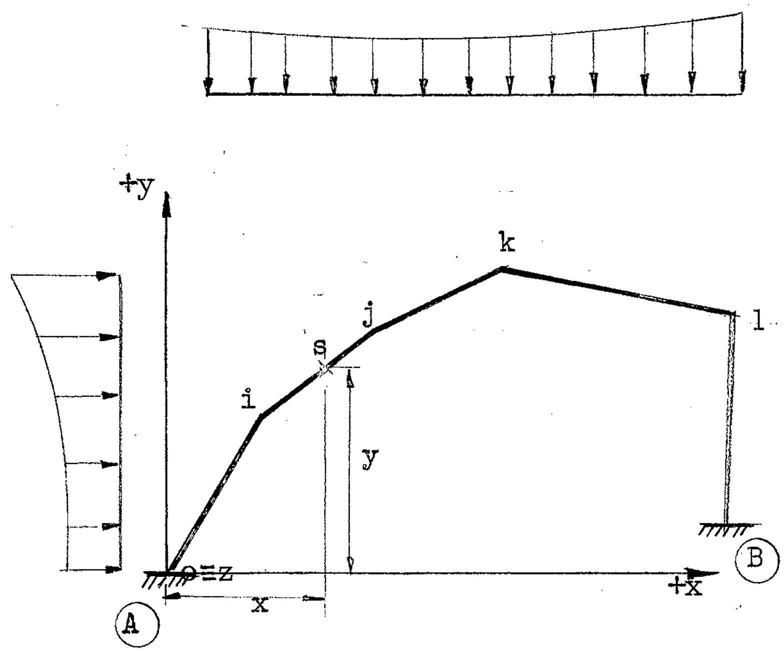


Figure 1-4. Real Frame - Real Loads

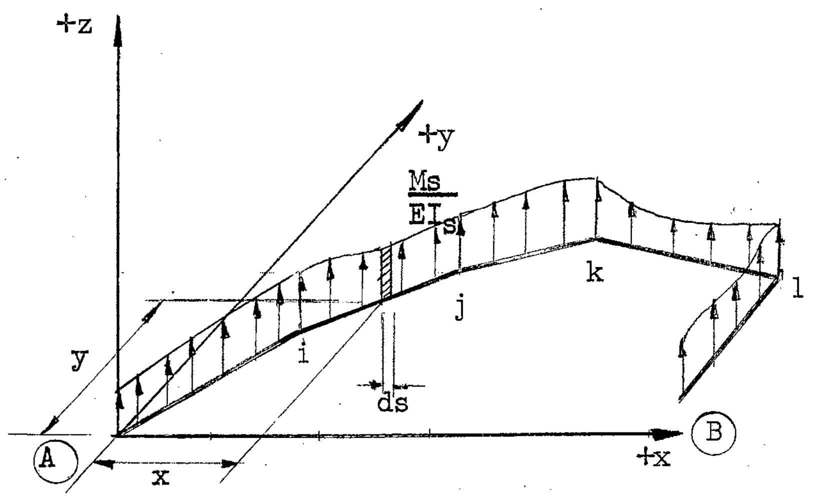


Figure 1-5. Conjugate Frame - Elemental Elastic Weights

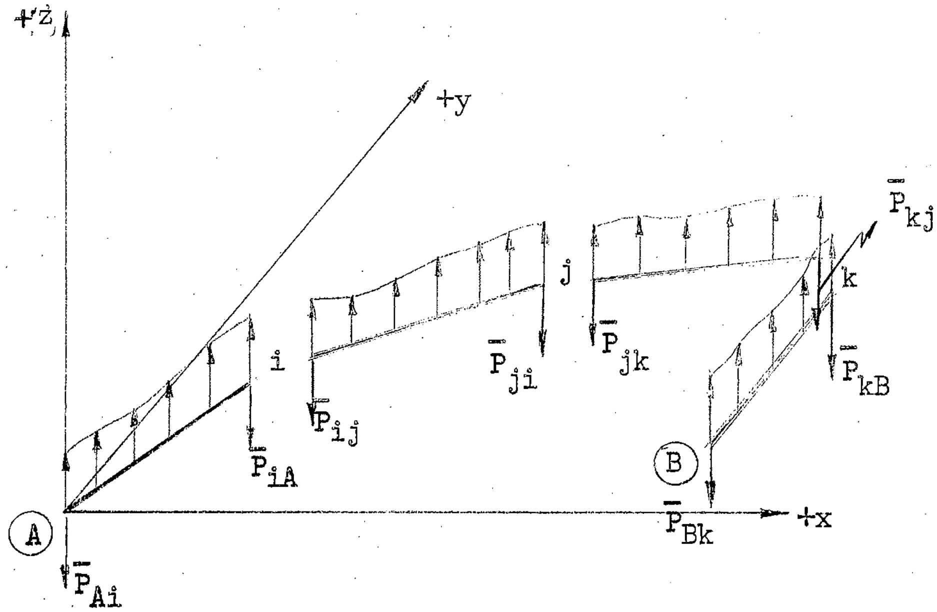


Figure 1-6. Conjugate Frame - Segmental Elastic Weights

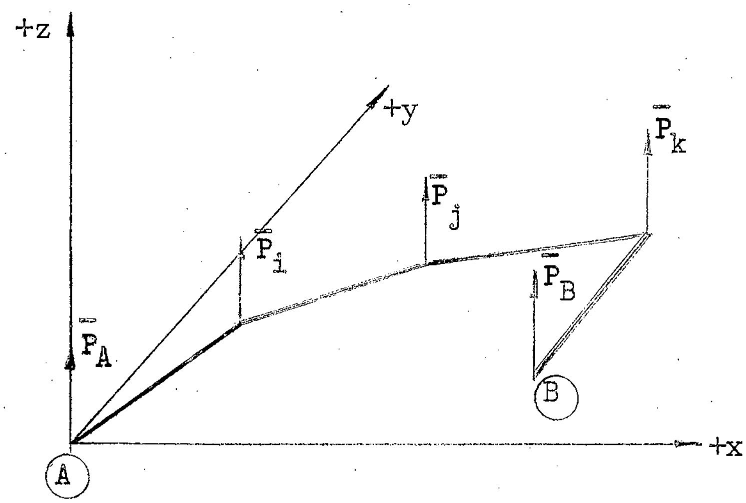


Figure 1-7. Conjugate Frame - Joint Elastic Weights

are in terms of three moments M_i, M_j, M_k. If more than two members are connected at one joint, the joint elastic weight must be written in terms of four moments. Two of each corresponding to one member. This simply states that the end bending moments (at a joint of a closed panel in a continuous or complex frame) are not necessarily equal. The joint elastic weights for the isolated part of the wedged frame (Figure 1-8a) are:

$$\begin{aligned} \bar{P}_j^{(ijk)} &= M_{ij}G_{ij} + M_{ji}F_{ji} + \tau_{ji} \\ & \quad M_{kj}G_{kj} + M_{jk}F_{jk} + \tau_{jk} \\ \bar{P}_j^{(kji)} &= M_{kj}G_{kj} + M_{jk}F_{jk} + \tau_{jk} \\ & \quad M_{lj}G_{lj} + M_{jl}F_{jl} + \tau_{jl} \end{aligned} \qquad (1\text{-}9b)$$

The graphical representation of the real frame (Figure 1-8a) and of the conjugate frame (Figure 1-8b) are shown.

The functions of Equation (1-9b) are:

<u>The Angular flexibility F_{ij}</u> is the end slope of a simple beam $\underline{ij}$ at j due to unit moment applied at the end j. (Figure 1-9a).

$$F_{ij} = \int_i^j \frac{u^2 du}{L_j^2 EI_u} \qquad F_{jk} = \int_i^j \frac{v^2 dv}{L_k^2 EI_v} \qquad (1\text{-}10a)$$

<u>The Carry-Over Value G_{ij}</u> is the end slope of a simple beam $\underline{ij}$ at i due to unit moment applied at the far end j (Figure 1-9a).

$$G_{ij} = \int_i^j \frac{u'u\ du}{L_j^2 EI_u} \qquad G_{jk} = \int_i^j \frac{v'v\ dv}{L_k^2 EI_v} \qquad (1\text{-}10b)$$

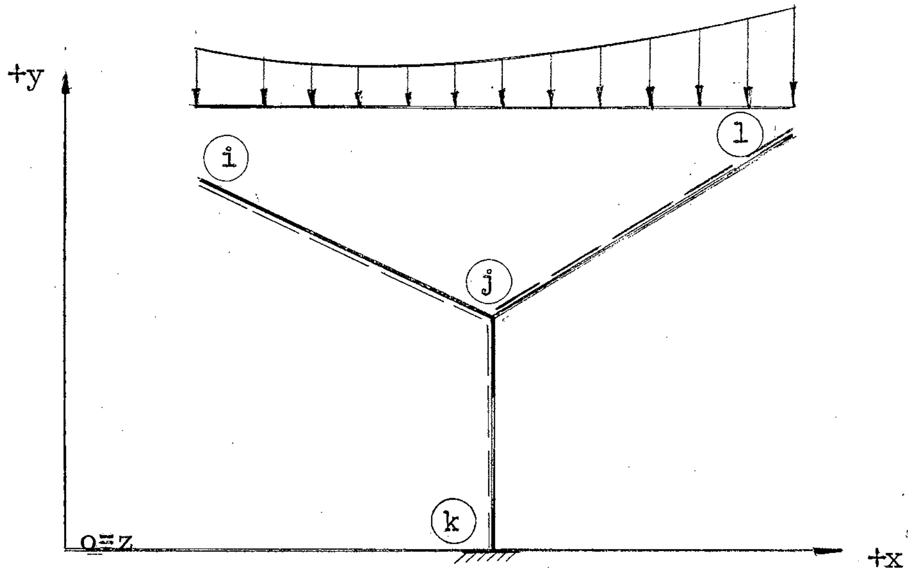


Figure 1-8a. Real Frame

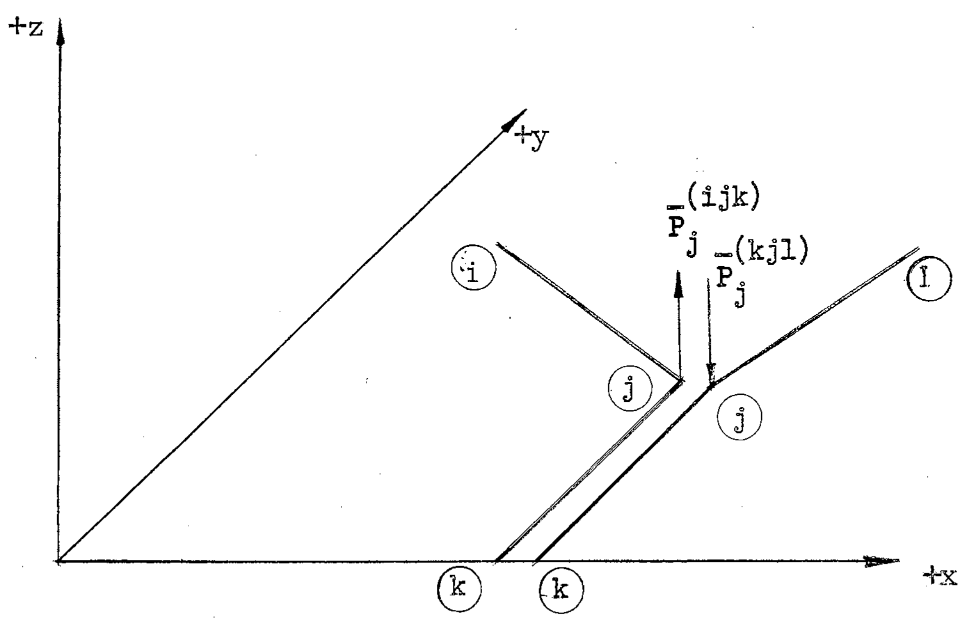


Figure 1-8b. Frame Joint Elastic Weights

The Angular Load Function τ_{ji} is the end slope of a simple beam ij at j due to loads (Figure 1-9b).

$$\tau_{ji} = \int_i^j \frac{BM_u u du}{L_j EI_u} \qquad \tau_{jk} = \int_j^K \frac{BM_v v dv}{L_k EI_v} \qquad (1\text{-}10c)$$

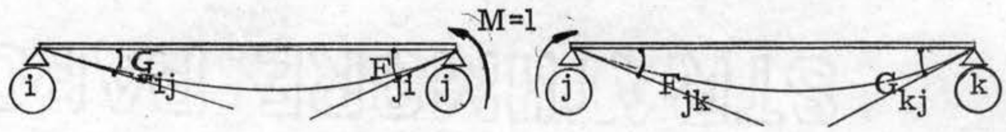


Figure 1-9a. Angular Flexibilities and Carry-Over Values

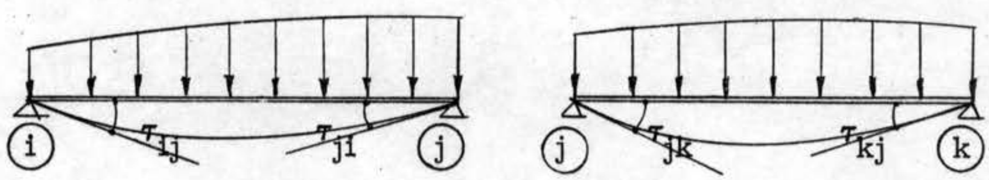


Figure 1-9b. Angular Load Functions

1-6. Interpretation of End Conditions

The interpretation of various end conditions of the real frame in terms of the end conditions of the conjugate frame are discussed in this part of the thesis.

(a) Two Hinge Frame

A frame with hinges at A and B (Figure 1-10a) developes:

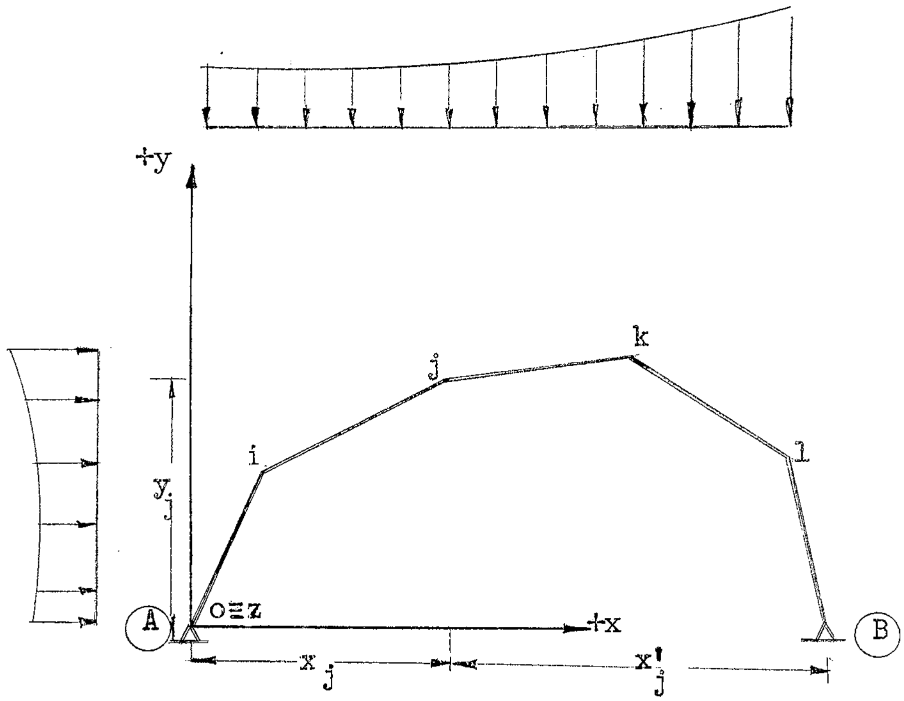


Figure 1-10a. Two Hinge Frame

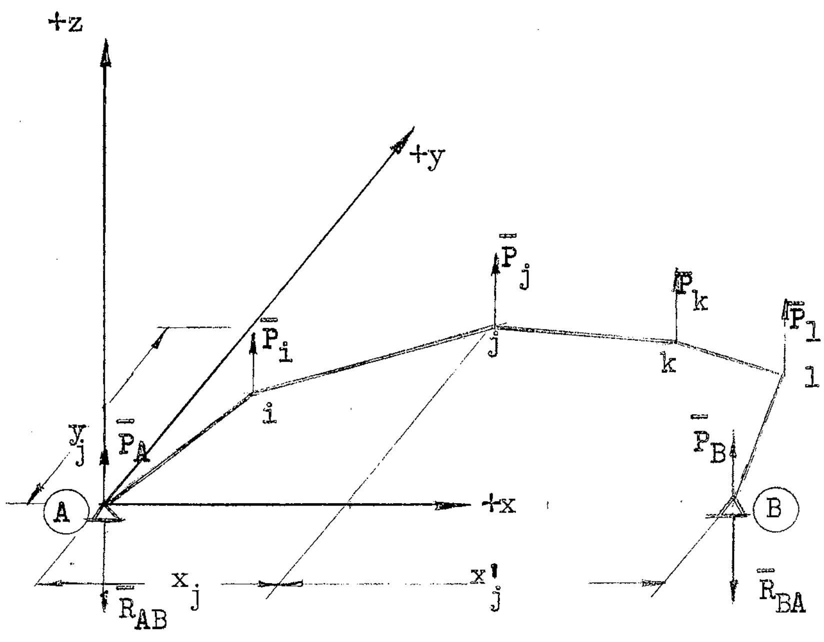


Figure 1-10b. Two Hinge Frame - Conjugate Frame

$$\theta_{AB} = \bar{R}_{AB} \qquad \theta_{BA} = \bar{R}_{BA}$$

$$\Delta_{ABX} = 0 \qquad \Delta_{BAX} = 0$$

$$\Delta_{ABY} = 0 \qquad \Delta_{BAY} = 0$$

The conjugate reactions are (Figure 1-10b)

$$\left.\begin{array}{ll} \bar{R}_{AB} = \sum_{A}^{B} \bar{P}_j \dfrac{x_j'}{L} & \bar{R}_{BA} = \sum_{A}^{B} \bar{P}_j \dfrac{x_j}{L} \\ \bar{M}_{ABX} = 0 & \bar{M}_{BAX} = 0 \\ \bar{M}_{ABY} = 0 & \bar{M}_{BAY} = 0 \end{array}\right\} \quad (1\text{-}11)$$

(b) <u>Fixed End Frame</u> (Figure 1-11a)

A frame fixed at A and B developes:

$$\left.\begin{array}{ll} \theta_{AB} = 0 & \theta_{BA} = 0 \\ \Delta_{ABX} = 0 & \Delta_{BAX} = 0 \\ \Delta_{ABY} = 0 & \Delta_{BAY} = 0 \end{array}\right\} \quad (1\text{-}12)$$

The conjugate reactions are (Figure 1-11b)

$$\bar{R}_{AB} = 0 \qquad \bar{R}_{BA} = 0$$

$$\bar{M}_{ABX} = 0 \qquad \bar{M}_{BAX} = 0$$

$$\bar{M}_{ABY} = 0 \qquad \bar{M}_{BAY} = 0$$

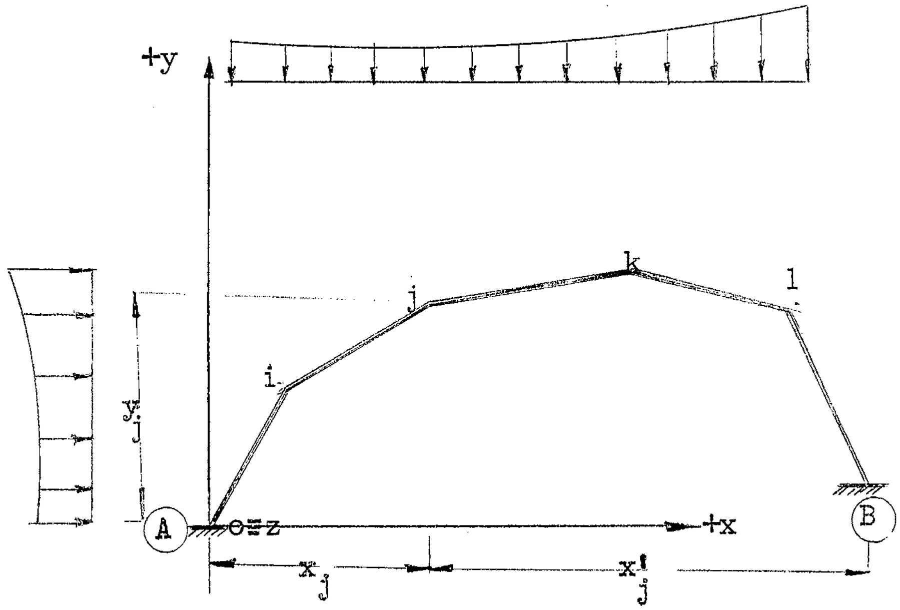


Figure 1-11a. Fixed End Frame

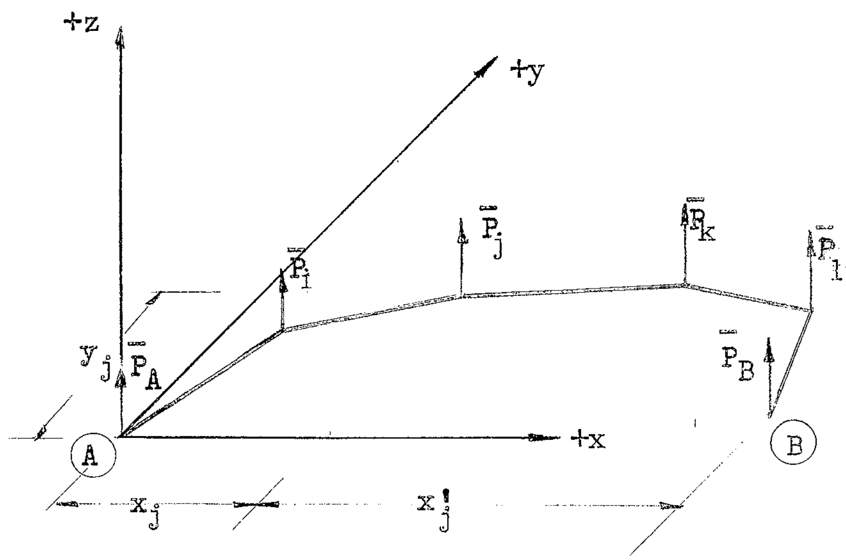


Figure 1-11b. Fixed End Frame - Conjugate Frame

(c) <u>Simply Supported Frame</u> (Figure 1-12a)

A frame with a roller at A and a hinge at B develops:

$$\theta_{AB} = \bar{R}_{AB} \qquad \theta_{BA} = \bar{R}_{BA}$$

$$\Delta_{ABX} = \cancel{0}\ \bar{M}_{ABX} \qquad \Delta_{BAX} = 0$$

$$\Delta_{ABY} = 0 \qquad \Delta_{BAY} = 0$$

The conjugate reactions are (Figure 1-12b)

$$\left.\begin{array}{ll} \bar{R}_{AB} = \sum_{A}^{B} \bar{P}_j \dfrac{x'_j}{L} & \bar{R}_{BA} = \sum_{A}^{B} \bar{P}_j \dfrac{x_j}{L} \\ \bar{M}_{ABX} = \sum_{A}^{B} \bar{P}_j y_i & \bar{M}_{BAX} = 0 \\ \bar{M}_{ABY} = 0 & \bar{M}_{BAY} = 0 \end{array}\right\} \quad (1\text{-}13)$$

(d) <u>Base Guided and Fixed</u> (Figure 1-13a)

A frame guided at A and fixed at B developes:

$$\theta_{AB} = 0 \qquad \theta_{BA} = 0$$

$$\Delta_{ABX} = \bar{M}_{ABX} \qquad \Delta_{BAX} = 0$$

$$\Delta_{ABY} = 0 \qquad \Delta_{BAY} = 0$$

The conjugate reactions are (Figure 1-13b)

$$\bar{R}_{AB} = 0 \qquad \bar{R}_{BA} = 0$$

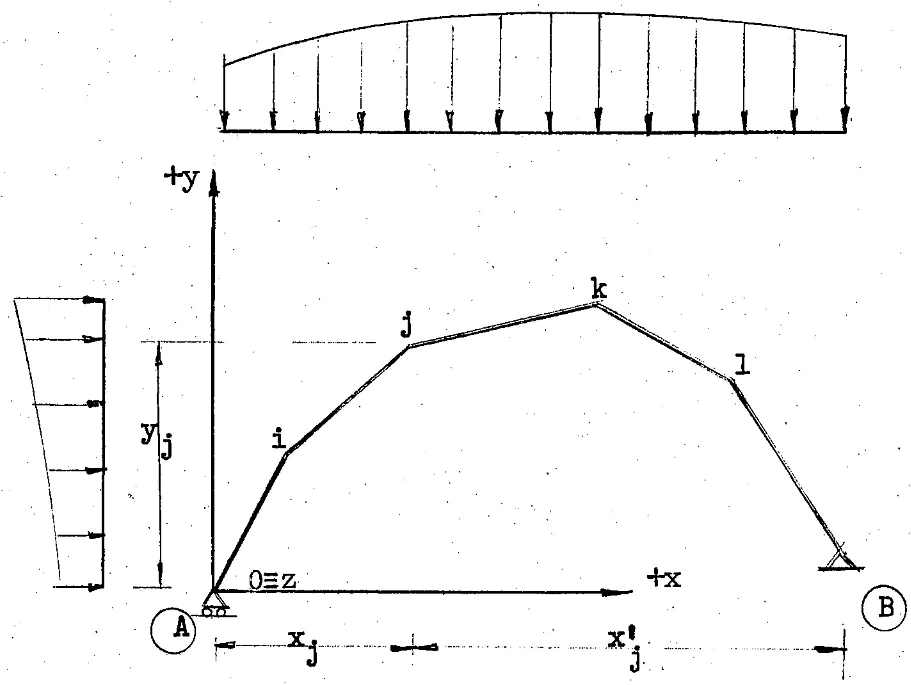


Figure 1-12a. Simply Supported Frame

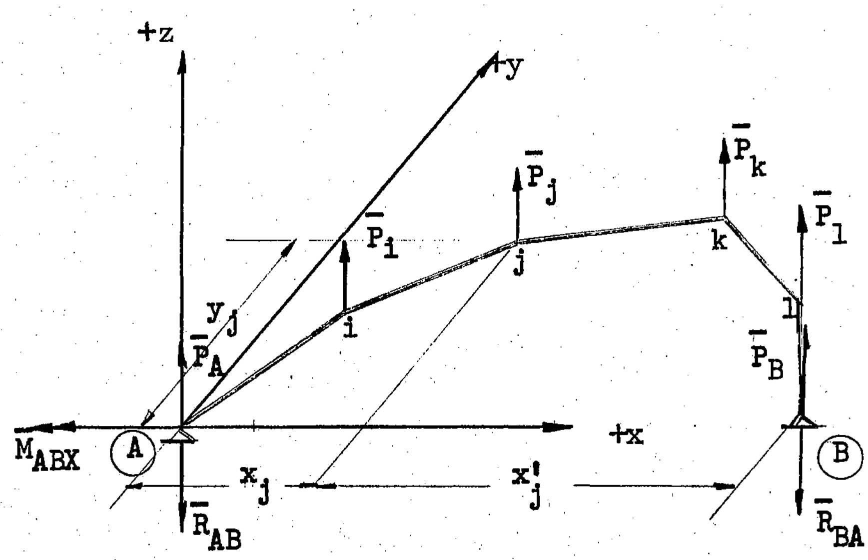


Figure 1-12b. Simply Supported Frame - Conjugate Frame

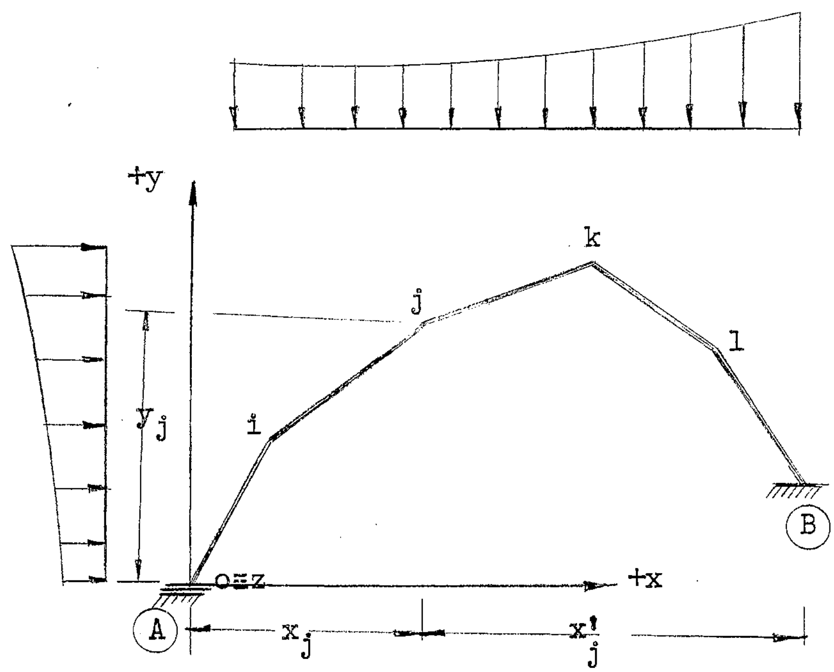


Figure 1-13a. Frame Guided and Fixed

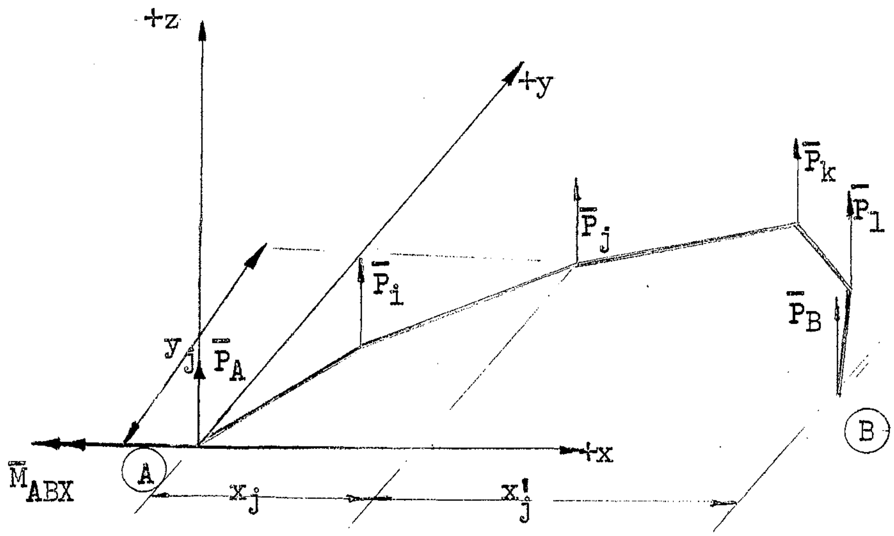


Figure 1-13b. Frame Guided and Fixed -
Conjugate Frame

$$\left.\begin{array}{ll} \bar{M}_{ABX} = \sum_{A}^{B} \bar{P}_j y_i & \bar{M}_{BAX} = 0 \\ \bar{M}_{ABY} = 0 & \bar{M}_{BAY} = 0 \end{array}\right\} \quad (1\text{-}14)$$

1-6. <u>Intermediate Hinge</u>

(a) <u>Fixed End Frame Hinged at the Middle</u> (Figure 1-14a)

A frame fixed at A and B and hinged at C developes:

$$\theta_{AC} = 0 \qquad \theta_{BC} = 0$$

$$\theta_{CA} = \bar{R}_{CA} \qquad \theta_{CB} = \bar{R}_{CB}$$

$$\Delta_{ACX} = 0 \qquad \Delta_{BCX} = 0$$

$$\Delta_{ACY} = 0 \qquad \Delta_{BCY} = 0$$

$$\Delta_{CAX} = \bar{M}_{CAX} \qquad \Delta_{CBX} = \bar{M}_{CBX}$$

$$\Delta_{CAY} = \bar{M}_{CAY} \qquad \Delta_{CBY} = \bar{M}_{CBY}$$

From the conditions of compatibility

$$\left.\begin{array}{l} \Delta_{CAX} + \Delta_{CBX} = 0 \\ \Delta_{CAY} + \Delta_{CBY} = 0 \end{array}\right\} \quad (1\text{-}15)$$

The conjugate moments are (Figure 1-14b)

$$\left.\begin{array}{l} \bar{M}_{CAX} + \bar{M}_{CBX} = \sum_{A}^{B} \bar{P}_j \bar{y}_j = 0 \\ \bar{M}_{CAY} + \bar{M}_{CBY} = \sum_{A}^{B} \bar{P} \bar{x}_j = 0 \end{array}\right\} \quad (1\text{-}16)$$

(b) Three Hinge Frame (Figure 1-15a)

A frame hinged at A, B and C developes:

$$\theta_{AC} = \bar{\bar{R}}_{AC} \qquad \theta_{BC} = \bar{\bar{R}}_{BC}$$

$$\theta_{CA} = \bar{\bar{R}}_{CA} \qquad \theta_{CB} = \bar{\bar{R}}_{CB}$$

$$\theta_{ACX} = 0 \qquad \theta_{BCX} = 0$$

$$\Delta_{CAX} = \bar{\bar{M}}_{CAX} \qquad \Delta_{CBX} = \bar{\bar{M}}_{CBX}$$

$$\Delta_{CAY} = \bar{\bar{M}}_{CAY} \qquad \Delta_{CBY} = \bar{\bar{M}}_{CBY}$$

$$\Delta_{ACY} = 0 \qquad \Delta_{BCY} = 0$$

From the conditions of compatibility

$$\begin{aligned} \Delta_{CAX} + \Delta_{CBX} &= 0 \\ \Delta_{CAY} + \Delta_{CBY} &= 0 \end{aligned} \tag{1-17}$$

The conjugate reactions are (Figure 1-16a, 1-16b)

$$\begin{aligned} \bar{\bar{R}}_{AC} &= \sum_{A}^{B} \bar{\bar{P}}_j \frac{\bar{\bar{u}}_j}{\bar{\bar{u}}_A} \\ \bar{\bar{R}}_{BC} &= \sum_{A}^{B} P_j \frac{\bar{\bar{v}}_j}{\bar{\bar{v}}_B} \end{aligned} \tag{1-18}$$

The conjugate moments are (Figure 1-15b)

$$\begin{aligned} \bar{\bar{M}}_{CAX} + \bar{\bar{M}}_{CBX} &= \bar{\bar{R}}_{AC}\bar{\bar{y}}_A + \bar{\bar{R}}_{BC}\bar{\bar{y}}_B - \sum_{A}^{B} \bar{\bar{P}}_j\bar{\bar{y}}_j \\ \bar{\bar{M}}_{CAY} + \bar{\bar{M}}_{CBY} &= \bar{\bar{R}}_{AC}\bar{\bar{x}}_A - \sum_{A}^{B} \bar{\bar{P}}_j\bar{\bar{x}}_j + \bar{\bar{R}}_{BC}\bar{\bar{x}}_B \end{aligned} \tag{1-19}$$

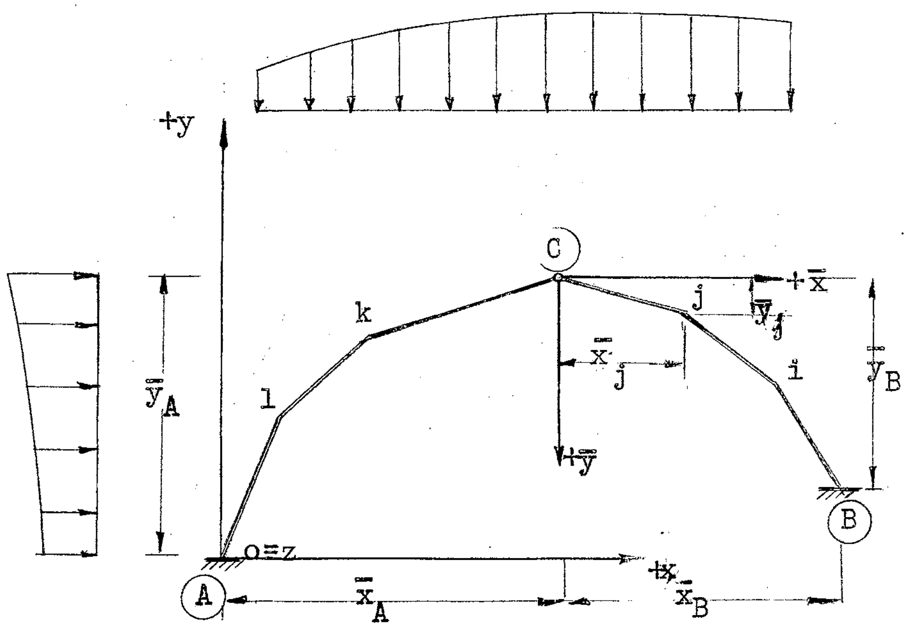


Figure 1-14a. Fixed Frame Hinged at the Middle

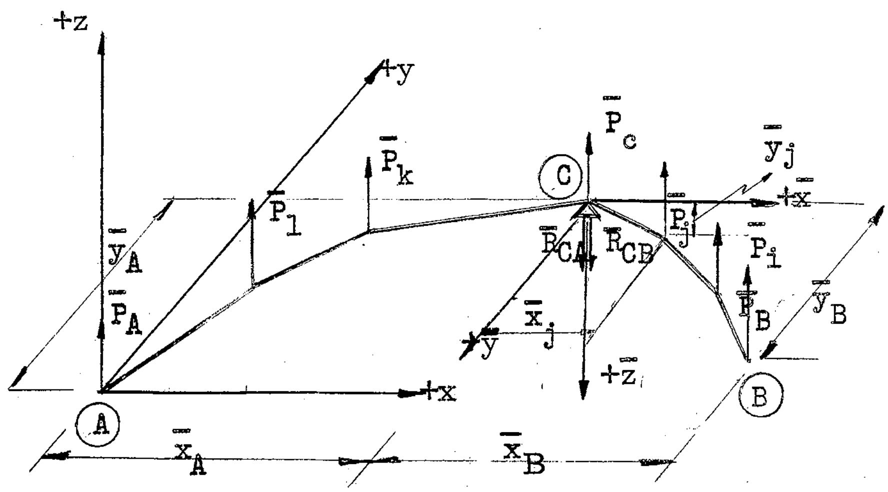


Figure 1-14b. Fixed Frame Hinged at the Middle - Conjugate Frame

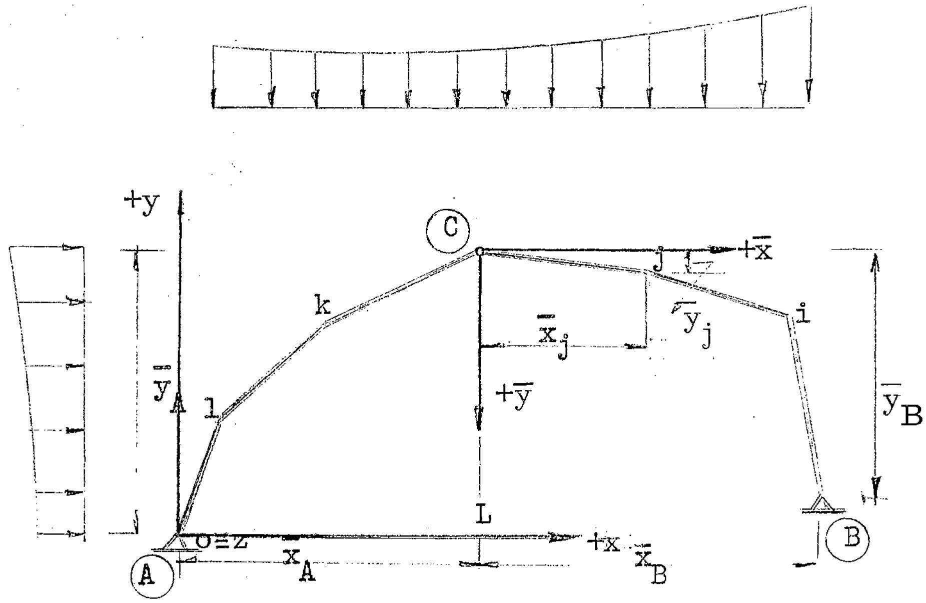


Figure 1-15a. Three Hinge Frame

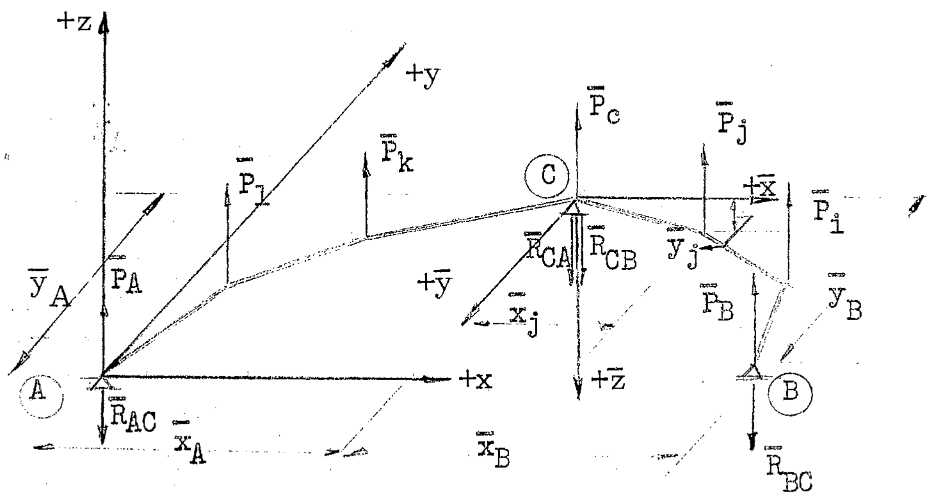


Figure 1-15b. Three Hinge Frame - Conjugate Frame

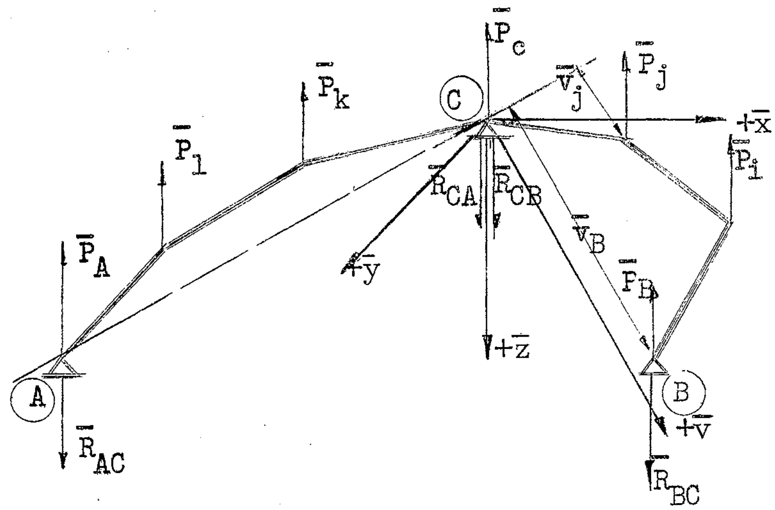


Figure 1-16a. Determination of Conjugate Reaction at B

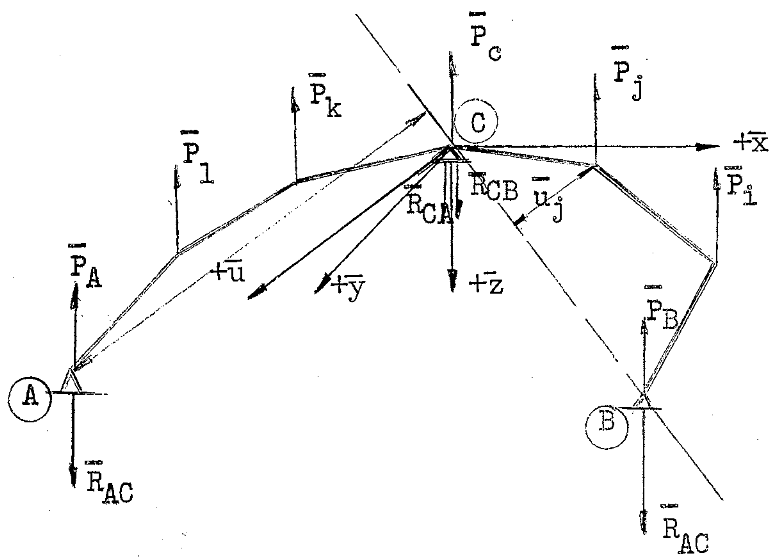


Figure 1-16b. Determination of Conjugate Reaction at A

CHAPTER II

CONTINUOUS WEDGED FRAME WITH BASE FIXED

2-1. Elasto-Static Equations

A continuous frame with base fixed shown in Figure 2-1 is considered. The redundants are selected as forces at hinges k and n, respectively. Once the redundants are selected, the bending moments of this continuous frame can be calculated by statics. In the calculation of bending moments, the following sign convention is established. Moments causing tension on the dotted side are positive. Moments causing tension of the opposite side are negative.

Because each panel is a closed string polygon, the structure is resolved in two conjugate frames as shown in Figure 2-2. To facilitate the use of joint elastic weights, the bending moments are calculated first in terms of redundants. The bending moments for the first panel ijklm are:

$$\left.\begin{aligned}
M_i &= X_1 \bar{y}_{ki} - Y_1 \bar{x}_{ki} - BM_i \\
M_j &= X_1 \bar{y}_{kj} - Y_1 \bar{x}_{kj} - BM_j \\
M_{1k} &= X_1 \bar{y}_{kl} - Y_1 \bar{x}_{kl} - BM_{1k} \\
M_{1m} &= X_1 \bar{y}_{kl} - Y_1 \bar{x}_{kl} - X_2 \bar{y}_{nl} + Y_2 \bar{x}_{nl} - BM_{1k} + BM_{1n}
\end{aligned}\right\} \quad (2\text{-}1)$$

$$M_m = X_1\bar{y}_{km} - Y_1\bar{x}_{km} - X_2\bar{y}_{nm} + Y_2\bar{x}_{nm} - BM_{mk} + BM_{mn}$$

The joint elastic weights for the first panel are:

$$\left.\begin{aligned}
\bar{P}_i &= M_j G_{ij} + M_i F_{ij} + \Sigma\tau_i \\
\bar{P}_j &= M_i G_{ij} + M_j(F_{ji} + F_{jk}) + \Sigma\tau_j \\
\bar{P}_l^{(klm)} &= M_m G_{lm} + M_{lm} F_{lm} + M_{lk} F_{lk} + \Sigma\tau_{klm} \\
\bar{P}_m &= M_{lm} G_{lm} + M_m F_{ml} + \Sigma\tau_m
\end{aligned}\right] \quad (2\text{-}2)$$

Once the joint elastic weights are available, the elasto-static equations can be written with respect to the hinge k.

$$\left.\begin{aligned}
\sum \bar{M}_{\bar{x}_k} = 0; \quad & \bar{P}_i\bar{y}_{ki} + \bar{P}_j\bar{y}_{kj} + \bar{P}_l^{(klm)}\bar{y}_{kl} + \bar{P}_m\bar{y}_{km} = 0 \\
\sum \bar{M}_{\bar{y}_k} = 0; \quad & \bar{P}_i\bar{x}_{ki} + \bar{P}_j\bar{x}_{kj} + \bar{P}_l^{(klm)}\bar{x}_{kl} + \bar{P}_m\bar{x}_{km} = 0
\end{aligned}\right] \quad (2\text{-}3)$$

Because there are only two redundants in each panel, two elasto-static equations are necessary. The same operation must be repeated for the second panel mlnop and two similar elasto-static equations must be written.

If the continuous frame has more than two panels, each panel introduces two additional redundants, and there are two additional elasto-static equations added.

Thus, there are as many elasto-static equations as redundants.

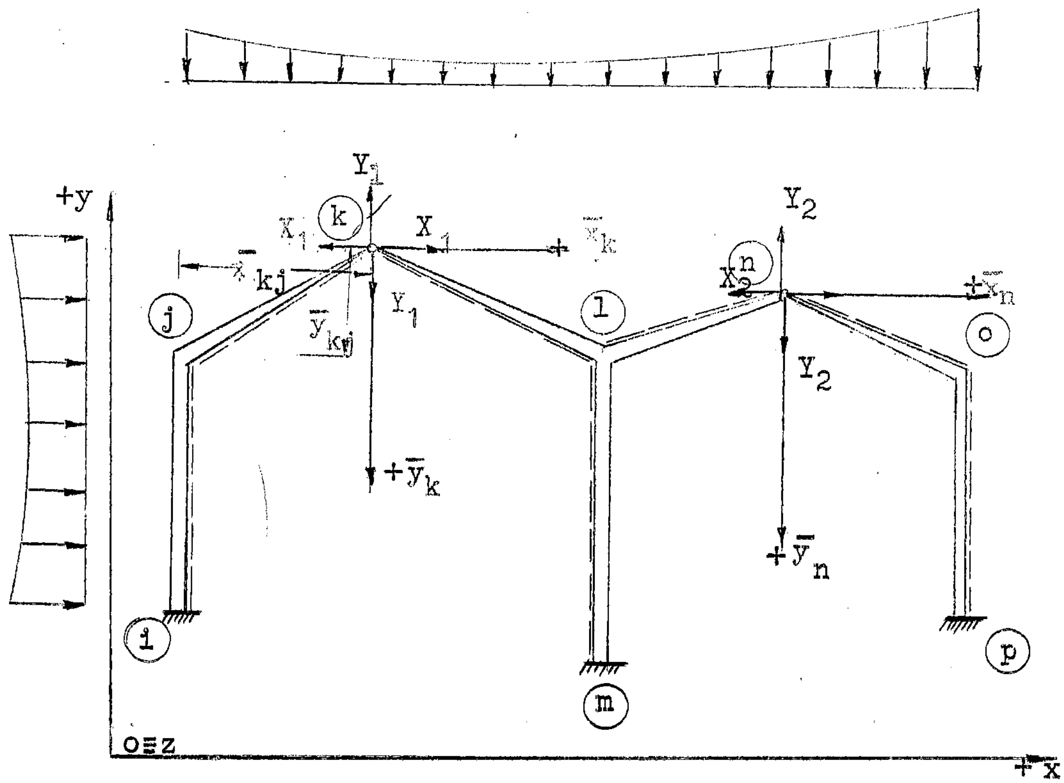


Figure 2-1. Continuous Wedged Frame with Base Fixed

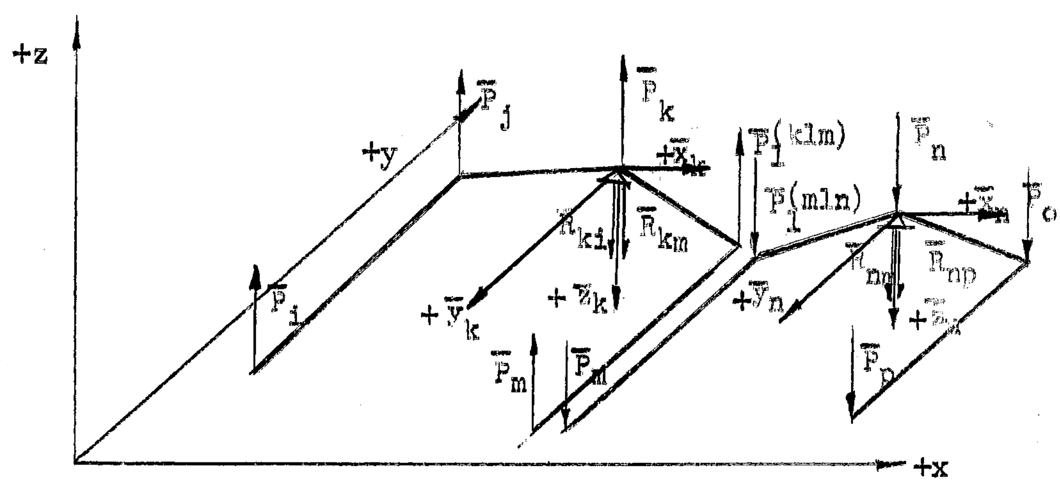


Figure 2-2 Conjugate Frame - Joint Elastic Weights

2-2. Force Matrix

The elasto-static equations yield a set of linear simultaneous equations which can be solved by classical methods or by successive approximations. The force matrix is shown in Table 2-1.

TABLE 2-1

FORCE MATRIX

$$\begin{bmatrix} C_{11} & C_{21} & C_{31} & C_{41} \\ C_{12} & C_{22} & C_{32} & C_{42} \\ C_{13} & C_{23} & C_{33} & C_{43} \\ C_{14} & C_{24} & C_{34} & C_{44} \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \end{bmatrix} = \begin{bmatrix} C_{51} \\ C_{52} \\ C_{53} \\ C_{54} \end{bmatrix}$$

CHAPTER III

CONTINUOUS WEDGED FRAME WITH BASE HINGED

3-1 Elasto-Static Equations

A continuous wedged frame with base hinged shown in Figure 3-1 is considered. The redundants are selected as forces at hinges k and n, respectively. Once the redundants are selected, the bending moments of this continuous frame can be calculated by statics. In the calculation of bending moments, the following sign convention is established. Moments causing tension on the dotted side are positive. Moments causing tension on the opposite side are negative.

Because each panel is a closed string polygon, the structure is resolved in two conjugate frames as shown in Figure 3-2. To facilitate the use of joint elastic weights, the bending moments are calculated first in terms of the redundants. The bending moments for the first panel <u>ijklm</u> are:

$$\left.\begin{aligned} M_j &= X_1\bar{y}_{kj} - Y_1\bar{x}_{kj} - BM_j \\ M_{lk} &= X_1\bar{y}_{ki} - Y_1\bar{x}_{kl} - BM_{lk} \\ M_{lm} &= X_1\bar{y}_{kl} - Y_1\bar{x}_{kl} - X_2\bar{y}_{nl} + Y_2\bar{x}_{nl} - BM_{lk} + BM_{ln} \end{aligned}\right] \quad (3\text{-}1)$$

Three of the redundants can be eliminated by the base conditions.

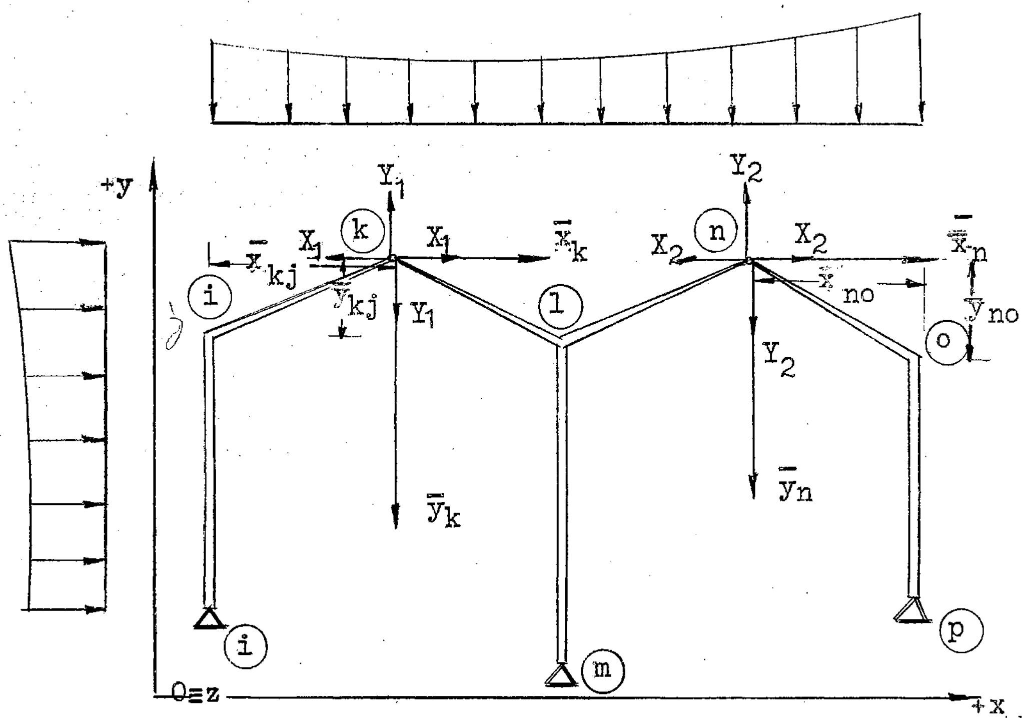


Figure 3-1. Continuous Wedged Frame with Base Hinged

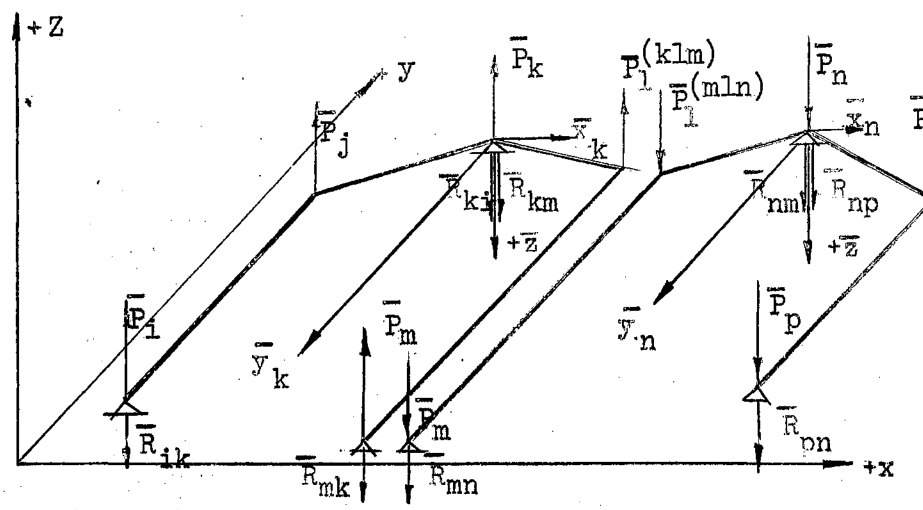


Figure 3-2. Conjugate Frames - Joint Elastic Weights

$$\sum M_i = 0; \qquad X_1\bar{y}_{ki} - Y_1\bar{x}_{ki} - BM_i = 0$$

$$Y_1 = \frac{X_1\bar{y}_{ki} - BM_i}{\bar{x}_{ki}} \tag{3-2a}$$

$$\sum M_p = 0; \qquad X_2\bar{y}_{np} - Y_2\bar{x}_{np} - BM_p = 0$$

$$Y_2 = \frac{X_2\bar{y}_{np} - BM_p}{\bar{x}_{np}} \tag{3-2b}$$

$$\sum M_m = 0; \qquad X_1\bar{y}_{km} - Y_1\bar{x}_{km} - X_2\bar{y}_{nm} + Y_2\bar{x}_{nm} - BM_{km} + BM_{nm} = 0$$

Substitutes Y_1, and Y_2 into the equation

$$X_2 = \frac{\bar{x}_{np}\left(X_1\left(\bar{y}_{km} - \frac{\bar{x}_{km}}{\bar{x}_{ki}}\bar{y}_{ki}\right) + BM_i\frac{\bar{x}_{km}}{\bar{x}_{ki}} - BM_{km} + BM_{nm} - BM_p\frac{\bar{x}_{nm}}{\bar{x}_{np}}\right)}{(\bar{y}_{nm}\bar{x}_{np} - \bar{x}_{nm}\bar{y}_{np})} \tag{3-2c}$$

$$Y_2 = \frac{\bar{y}_{np}\left(X_1\left(\bar{y}_{km} - \frac{\bar{x}_{km}}{\bar{x}_{ki}}\bar{y}_{ki}\right) + BM_i\frac{\bar{x}_{km}}{\bar{x}_{ki}} - BM_{km} + BM_{nm} - BM_p\frac{\bar{x}_{nm}}{\bar{x}_{np}}\right)}{(\bar{y}_{nm}\bar{x}_{np} - \bar{x}_{nm}\bar{y}_{np})}$$

$$- \frac{BM_p}{\bar{x}_{np}} \tag{3-2d}$$

Then, the bending moments for the first panel become

$$M_j = X_1\left(\bar{y}_{kj} - \frac{\bar{x}_{kj}}{\bar{x}_{ki}}\bar{y}_{ki}\right) + BM_i\frac{\bar{x}_{kj}}{\bar{x}_{ki}} - BM_j$$

$$M_{lk} = X_l \left(\bar{y}_{kl} - \frac{\bar{x}_{kl}}{\bar{x}_{ki}}\bar{y}_{ki}\right) + BM_i \frac{\bar{x}_{kl}}{\bar{x}_{ki}} - BM_{lk}$$

Let

$$\frac{\bar{x}_{np}\bar{y}_{nl} - \bar{x}_{nl}\bar{y}_{np}}{\bar{y}_{nm}\bar{x}_{np} - \bar{x}_{nm}\bar{y}_{np}} = \mu \tag{3-3}$$

$$M_{lm} = X_l\left(\bar{y}_{kl} - \frac{\bar{x}_{kl}\bar{y}_{ki}}{\bar{x}_{ki}} - \mu\left(\bar{y}_{km} - \frac{\bar{y}_{ki}\bar{x}_{km}}{\bar{x}_{ki}}\right)\right)$$

$$+ BM_i\left(\frac{\bar{x}_{kl}}{\bar{x}_{ki}} - \mu\frac{\bar{x}_{km}}{\bar{x}_{ki}}\right) + BM_{km}\mu - BM_{nm}\mu + BM_p\left(\frac{\bar{x}_{nm}}{\bar{x}_{np}}\mu - \frac{\bar{x}_{nl}}{\bar{x}_{np}}\right)$$

$$-BM_{lk} + BM_{ln}$$

The joint elastic weights for the first panel are:

$$\left.\begin{aligned}
\bar{P}_i &= M_j G_{ij} + \Sigma\tau_i \\
\bar{P}_j &= M_j(F_{ji} + F_{jk}) + \Sigma\tau_j \\
\bar{P}_l^{(klm)} &= M_{im}F_{lm} + M_{lk}F_{lk} + \Sigma\tau_l^{(klm)}
\end{aligned}\right] \quad (3\text{-}4)$$

Once the joint elastic weights are available, the elastic reactions for the first panel can be found by Equation (1-18)

$$\left.\begin{aligned}
\bar{R}_{ik} &= \sum \bar{P}_j \frac{\bar{u}_j}{\bar{u}_j} \\
\bar{R}_{mk} &= \sum \bar{P}_j \frac{\bar{v}_j}{\bar{v}_m}
\end{aligned}\right] \quad (3\text{-}5)$$

The elasto-static equation can be written with respect to the hinge m.

$$\sum \bar{R}_m = 0; \qquad \bar{R}_{mk} + \bar{R}_{mn} = 0 \qquad (3\text{-}6)$$

Because there is only one redundant for a two panel continuous frame, one elasto-static equation is necessary.

If the continuous frame has more than two panels, each panel introduces one additional redundant, and there is one additional elasto-static equation added. Thus, there are as many elasto-static equations as redundants.

3-2. Force Matrix

The elasto-static equations yield a set of linear simultaneous equations which can be solved by classical methods or by successive approximation. The force matrix is shown in Table 3-1.

TABLE 3-1

FORCE MATRIX

$$\begin{bmatrix} C_{11} & C_{21} & - & - & - & C_{(n-1)_1} \\ C_{12} & C_{22} & - & - & - & C_{(n-1)_2} \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ C_{1(n-1)} & C_{2(n-1)} & - & - & - & C_{(n-1)(n-1)} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ - \\ - \\ - \\ X_{n-1} \end{bmatrix} = \begin{bmatrix} C_{n_1} \\ C_{n_2} \\ - \\ - \\ - \\ C_{n(n-1)} \end{bmatrix}$$

CHAPTER IV

BEAM CONSTANTS

4-1. General Notes and Assumptions

Beam constants for tapered beams of I-section, box section, and T-section are investigated in this chapter. Typical sections are shown in Figure 4-1.

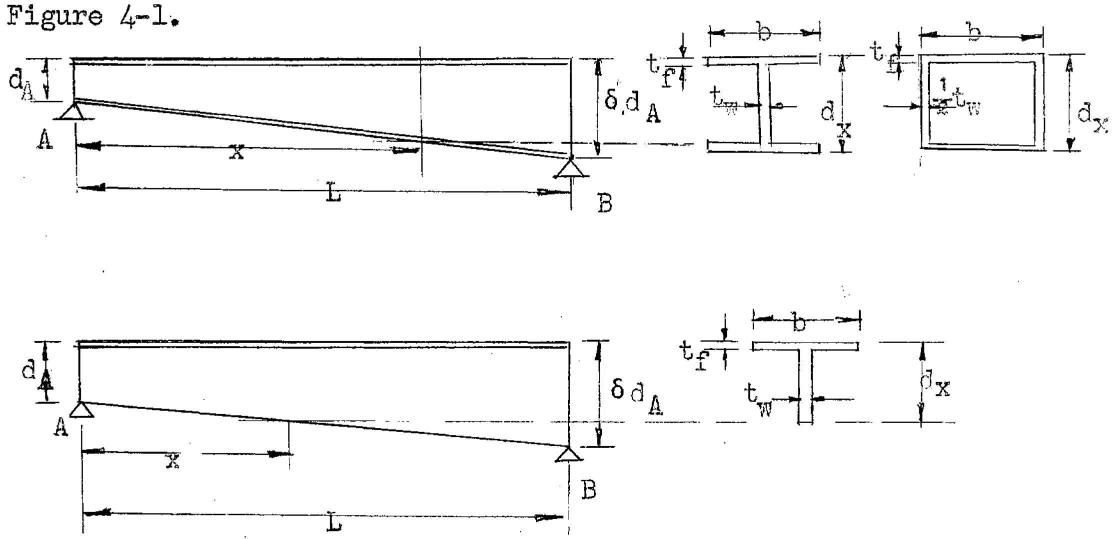


Figure 4-1. Typical Sections of Tapered Beam

The nomenclatures used in Figure 4-1 are:

b = The constant width of the section through the whole beam

t_f = The thickness of flange

t_w = The thickness of web of I-beam or T-beam or total thickness of two webs of box beam

d_A = The minimum depth of the beam

δd_A = The maximum depth of the beam

The depth of the beam at any point x from left end is:

$$d_x = d_A + \frac{(\delta - 1)\, d_{A} x}{L}$$
$$= d_A \left(1 + \psi \frac{x}{L}\right) \qquad (4\text{-}1)$$

Where:

$\psi = \delta - 1$

The moment of inertia for a tapered beam of I-section or box section at any point x is:

$$I_x = \frac{b d_A^3}{12}\left(1+\psi \frac{x}{L}\right)^3 - \frac{(b-t_w)\, d_A^3}{12}\left(1-2\frac{t_f}{d_A}+\frac{x}{L}\right)^3 \qquad (4\text{-}2)$$

The moment of inertia for a tapered beam of T-section at any point x is:

$$I_x = \frac{1}{12}\, b t_f^3 + \frac{1}{12}\, t_w \left(d_A\left(1+\psi \frac{x}{L}\right) - t_f\right)^3 + \frac{b t_f t_w d_A\left(1+\psi \frac{x}{L}\right)\left(d_A\left(1+\psi\frac{x}{L}\right)-t_f\right)}{4\left(b t_f + \left(d_A\left(1+\psi \frac{x}{L}\right)-t_f\right) t_f\right)} \qquad (4\text{-}3)$$

From Equations (4-2) and (4-3) it is obvious that the exact evaluation of the angular functions by integration will be difficult. In order to surmount this difficulty, the variation in the moment of inertia of the tapered beam can be represented as a power function:

$$I_x = I_o \left(1 + \psi \frac{x}{L}\right)^{\rho} \qquad (4\text{-}4)$$

This representation makes the analysis of tapered beams feasible, since the integrals in the equations of angular functions can now be evaluated. The exponent   in the equation is called shape factor, which can be determined by the cross section at the two ends.

$I_x = I_o$;   where $x = 0$

$I_x = I_B$;   where $x = L$

Since:

$$I_B = I_o(1 + \psi)^{\rho} = I_o\left(\frac{\delta d_A}{d_A}\right)^{\rho} \tag{4-5}$$

from which

$$\frac{\log I_B/I_o}{\log \delta} = \rho \tag{4-6}$$

Using this value of ρ in Equation (4-4), the exact values of the moment of inertia at both ends can be obtained. However, for intermediate points, there will be a slight deviation. The maximum error, as computed for a large number of beams was 5%. Usually, it is around 1%. (15)

The values of ρ is between 2.1 and 2.9.

4-2. Integral Functions of Tapered Beams

Through the process of computing the elastic constants for tapered beams, the following integral functions will frequently reoccur:

Defining:

$$a_1 = \frac{(1 + \psi)^{1-\rho} - 1}{1 - \rho} \qquad\qquad b_1 = \frac{(1 + n\psi)^{1-\rho} - 1}{1 - \rho}$$

$$a_2 = \frac{(1+\psi)^{2-\rho}-1}{2-\rho} \qquad b_1 = \frac{(1+n\psi)^{2-\rho}-1}{2-\rho}$$

$$a_3 = \frac{(1+\psi)^{3-\rho}-1}{3-\rho} \qquad b_3 = \frac{(1+n\psi)^{3-\rho}-1}{3-\rho} \qquad (4\text{-}7)$$

$$a_4 = \frac{(1+\psi)^{4-\rho}-1}{4-\rho} \qquad b_4 = \frac{(1+n\psi)^{4-\rho}-1}{4-\rho}$$

$$a_5 = \frac{(1+\psi)^{5-\rho}-1}{5-\rho} \qquad b_5 = \frac{(1+n\psi)^{5-\rho}-1}{5-\rho}$$

Let $x = tL$, $d_x = (1+\psi t)d_A$, $I_x = I_o(1+\psi t)^{\rho}$, $dx = Ldt$,

(4-8)

$$Q_1 = I_o\int_o^1 \frac{d_t}{I_t} = \frac{1}{\psi}(a_1)$$

$$Q_2 = I_o\int_o^1 \frac{tdt}{I_t} = \frac{1}{\psi^2}(a_2 - a_1)$$

$$Q_3 = I_o\int_o^1 \frac{t^2dt}{I_t} = \frac{1}{\psi^3}(a_3 - 2a_2 + a_1) \qquad (4\text{-}9)$$

$$Q_4 = I_o\int_o^1 \frac{t^3dt}{I_t} = \frac{1}{\psi^4}(a_4 - 3a_3 + 3a_2 - a_1)$$

$$Q_5 = I_o\int_o^1 \frac{t^4dt}{I_t} = \frac{1}{\psi^5}(a_5 - 4a_4 + 6a_3 - 4a_2 + a_1)$$

$$
\left.
\begin{aligned}
Q_{1n} &= I_o \int_o^n \frac{t\,dt}{I_t} = \frac{1}{\psi}\,(b_1) \\
Q_{2n} &= I_o \int_o^n \frac{t\,dt}{I_t} = \frac{1}{\psi^2}\,(b_2 - b_1) \\
Q_{3n} &= I_o \int_o^n \frac{t^2 dt}{I_t} = \frac{1}{\psi^3}\,(b_3 - 2b_2 + b_1) \\
Q_{4n} &= I_o \int_o^n \frac{t^3 dt}{I_t} = \frac{1}{\psi^4}\,(b_4 - 3b_3 + 3b_2 - b_1) \\
Q_{5n} &= I_o \int_o^n \frac{t^4 dt}{I_t} = \frac{1}{\psi^5}\,(b_5 - 4b_4 + 6b_3 - 4b_2 + b_1)
\end{aligned}
\right\} \quad (4\text{-}10)
$$

The existence and the physical meaning of these functions are shown in articles 4-3, 4-4, 4-5, and 4-6 of this thesis.

4-3. Flexibilities and Carry-Over Values

A tapered beam of I-section, Box section or T-section simply supported at the ends, is acted on by a unit moment applied at B (Figure 4-2).

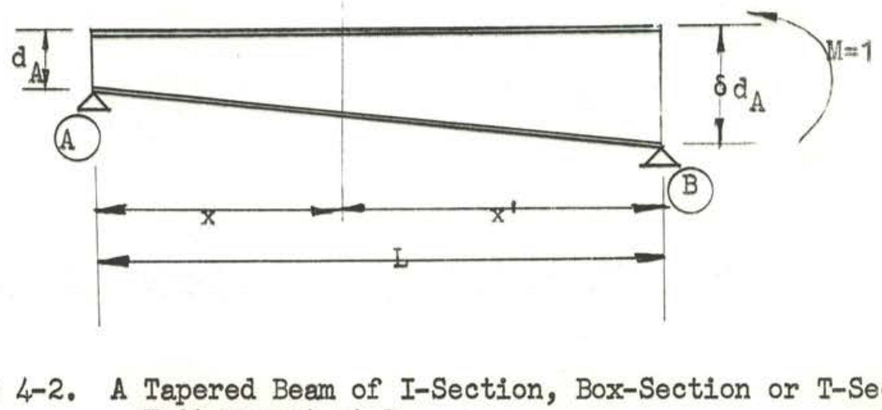


Figure 4-2. A Tapered Beam of I-Section, Box-Section or T-Section Unit Moment at B.

The angular flexibility (Equation 1-10a) is:

$$F_{BA} = \int_A^B \frac{x^2 dx}{L^2 E I_x} \tag{4-11a}$$

or in terms of t, I_o, I_t with notation Q's (Equation 4-10)

$$F_{BA} = \frac{L}{E\ I_o}\ (I_o \int_o^1 \frac{t^2 dt}{E\ I_t}) = \frac{L}{E\ I_o}\ Q_3 = \underline{\underline{f_2\ \frac{L}{E\ I_o}}} \tag{4-11}$$

The carry-over value (Equation 1-10b) is:

$$G_{AB} = \int_A^B \frac{(L - x)\ x dx}{L^2 E\ I_x} \tag{4-12a}$$

or in terms of t, I_o, I_t with notation Q's (Equation 4-10)

$$G_{AB} = \frac{L}{E\ I_o}\ (\ I_o \int_o^1 \frac{t dt}{E\ I_t} - I_o \int_o^1 \frac{t^2 dt}{E\ I_t}\ )$$

$$= \frac{L}{E\ I_o}\ (Q_2 - Q_3) = g\ \frac{L}{E\ I_o} \tag{4-12}$$

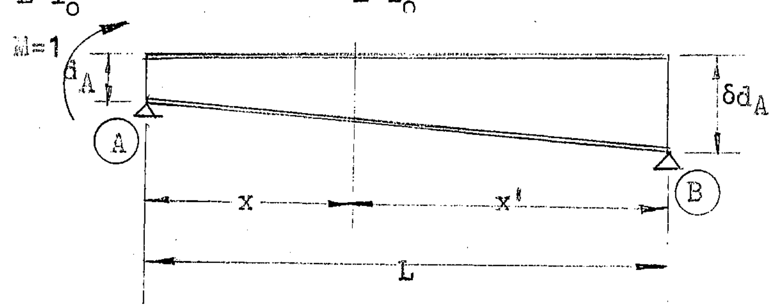


Figure 4-3. A Tapered Beam of I-Section, Box section or T-Section Unit Moment at A

If the unit moment applied at A (Figure 4-3), the angular flexibility (Equation 1-10b) is:

$$F_{AB} = \int_A^B \frac{(L-x)^2 dx}{L^2 E\ I_x} = \int_o^L \frac{dx}{EI_x} - 2\int_o^L \frac{xdx}{LEI_x} + \int_o^L \frac{x^2 dx}{L^2 EI_x} \tag{4-13a}$$

or in terms of t, I_o, I_t, with notation Q's (Equation 4-10)

$$F_{AB} = \frac{L}{EI_o}\left(I_o \int_o^1 \frac{dt}{EI_t} - 2\ I_o \int_o^1 \frac{tdt}{EI_t} + I_o \int_o^1 \frac{t^2 dt}{EI_t} \right)$$

$$= \frac{L}{EI_o}\left(Q_1 - 2Q_2 + Q_3 \right) = \underline{f_1 \frac{L}{EI_o}} \tag{4-13}$$

From Maxwell's theorem the carry-over value

$$G_{BA} = G_{AB} \tag{4-14}$$

4-4. Load Function - Unit Load

A tapered beam of I-section, box section or T-section, simply supported at the ends, is acted on by a unit concentrated load (Figure 4-4)

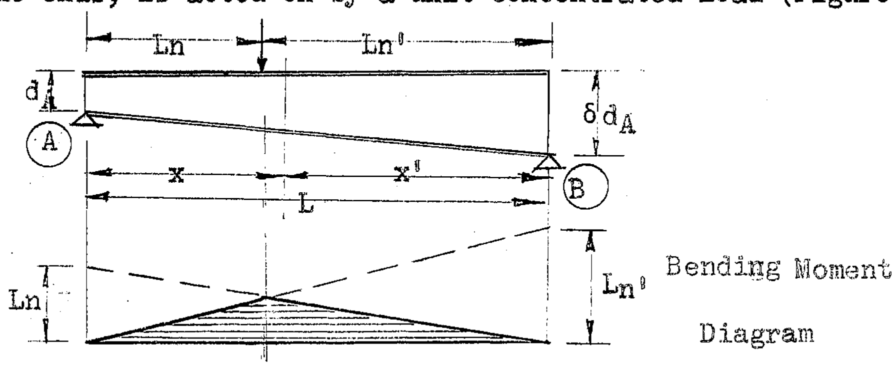


Figure 4-4. A Tapered Beam of I-Section, Box Section or T-Section Concentrated Load

The bending moment at x, due to a unit load at Ln is:

$$BM^{(A)}_{x = 0 \rightarrow Ln} = nx' - (x' - Ln')$$
$$BM^{(A)}_{x = Ln \rightarrow L} = nx' \qquad (4\text{-}15)$$

The end slope at B due to a unit load at Ln (Equation 1-10c) is:

$$\tau^{(LL)}_{BA} = \int_A^B \frac{BM_x xdx}{LEI_x} = \int_0^L \frac{nxx'dx}{LEI_x} - \int_0^{Ln} \frac{(x' - Ln')\ xdx}{LEI_x}$$
$$= \int_0^L \frac{nLxx'dx}{L^2EI_x} + \int_0^{Ln} \frac{x^2dx}{LEI_x} - \int_0^{Ln} \frac{nLxdx}{LEI_x} \qquad (1\text{-}16a)$$

or in terms of t, I_o, I_t, with notation Q's (Equations 4-9, 4-10)

$$\tau^{(LL)}_{BA} = G_{AB}nL - \frac{nL^2}{EI_o}\left(I_o\int_0^n \frac{tdt}{EI_t}\right) + \frac{L^2}{EI_o}\left(I_o\int_0^n \frac{t^2dt}{EI_t}\right)$$
$$= G_{AB}nL + \frac{L^2}{EI_o}(Q_{3n} - Q_{2n}n) \qquad (4\text{-}16b)$$
$$= \frac{L}{EI_o}(nQ_2 - nQ_3 - nQ_{2n} + Q_{3n})$$

$$= \underline{\underline{t_2 \frac{L^2}{EI_o}}} \qquad (1\text{-}16)$$

The end slope at A due to a unit load at Ln (Equation 1-10c) is:

$$\left.\begin{aligned}\tau_{AB}^{(LL)} &= \int_A^B \frac{BM_x x' dx}{LEI_x} = \int_A^B \frac{BM'_x (L-x)}{EI_x} \\ &= \int_A^B \frac{BM_x dx}{EI_x} - \int_A^B \frac{BM_x x dx}{EI_x} \\ &= \int_A^B \frac{nx' dx}{EI_x} - \int_A^B \frac{(x' - Ln') dx}{EI_x} - \tau_{BA}^{(LL)} \\ &= \int_o^L \frac{nL(x - x')x' dx}{L^2 EI_x} + \int_o^{Ln} \frac{x dx}{EI_x} - \int_o^{Ln} \frac{nL dx}{EI_x} - \tau_{BA}^{(LL)}\end{aligned}\right\} \quad (4\text{-}17a)$$

or in terms of t, I_o, I_t with notation Q's (Equations 4-9, 4-10)

$$\left.\begin{aligned}\tau_{AB}^{(LL)} &= Ln(F_{AB} + G_{AB}) + \frac{L^2}{EI_o}\left(I_o \int_o^n \frac{t dt}{EI_t} - I_o \int_o^n \frac{n dt}{EI_t}\right) - \tau_{BA}^{(LL)} \\ &= \frac{L^2}{EI_o}(nQ_1 - 2nQ_2 + nQ_3 - nQ_{1n} + (1 + n) Q_{2n} - Q_{3n})\end{aligned}\right\} \quad (4\text{-}17b)$$

$$= t_1 \frac{L^2}{EI_o} \qquad (4\text{-}17)$$

4-5. <u>Load Function - Uniform Load:</u>

A tapered beam of I-section, box section or T-section simply supported at the ends, is acted on by a uniformly distributed load of intensity w (Figure 4-5)

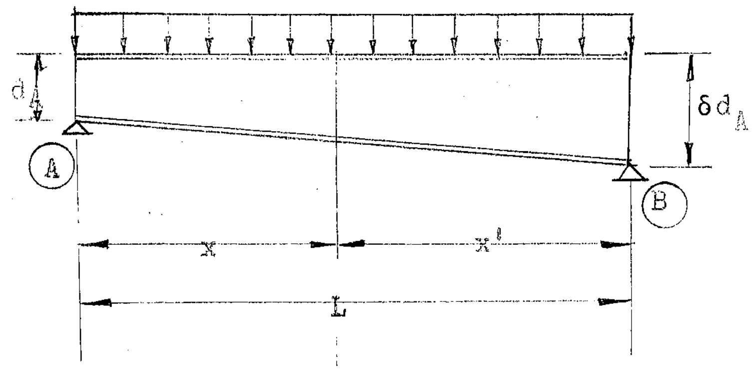


Figure 4-5. A Tapered Beam of I-Section, Box Section of T-Section Uniformly Distributed Load

The moment at x, due to uniformly distributed load

$$BM_x = \frac{1w}{2}(L - x)\,x$$

The right end slope (Equation 1-10c) is:

$$\tau_{BA}^{(UL)} = \int_A^B \frac{BM_x x dx}{EI_x}$$

$$= \int_o^L \frac{w(L - x)x^2 dx}{2L\ E\ I_x} = \frac{w}{2}\left(\int_o^L \frac{x^2 dx}{EI_x} - \int_o^L \frac{x^3 dx}{LEI_x}\right) \qquad (4\text{-}19a)$$

or in terms of t, I_o, I_t, with notation Q's (Equation 4-9)

$$\tau_{BA}^{(UL)} = \frac{L^3 w}{EI_o}\left(\frac{1}{2}\ I_o\int_o^1 \frac{t^2 dt}{EI_t} - \frac{1}{2}\ I_o\int_o^1 \frac{t^3 dt}{EI_t}\right)$$

$$= \frac{wL^3}{EI_o}\left(\frac{1}{2}\ Q_3 - \frac{1}{2}\ Q_4\right) \qquad (4\text{-}19b)$$

$$= t_4\ \frac{wL^3}{EI_o} \qquad (4\text{-}19)$$

The left end slope (Equation 1-10c)

$$
\left.
\begin{aligned}
\tau_{AB}^{(UL)} &= \int_A^B \frac{BM_x(L - x)\, dx}{LEI_x} \\
&= \int_A^B \frac{1w}{2}\left(\frac{(L - x)^2 x dx}{LEI_x}\right. \\
&= \int_o^L \frac{w}{2}\left(\frac{Lx\, dx}{EI_x} - \int_o^L \frac{w\, x^2 dx}{EI_x} + \int_o^L \frac{w\, x^3 dx}{2\, LEI_x}\right.
\end{aligned}
\right\} \quad (4\text{-}20a)
$$

or in terms of t, I_o, I_t with notation Q's (Equation 4-9)

$$
\left.
\begin{aligned}
\tau_{AB}^{(UL)} &= \frac{wL^3}{EI_o}\left(\frac{1}{2}\, I_o \int_o^1 \frac{tdt}{EI_t} - I_o \int_o^1 \frac{t^2 dt}{EI_t} + \frac{I_o}{2}\int_o^1 \frac{t^3 dt}{EI_t}\right. \\
&= \frac{wL^3}{EI_o}\left(\frac{1}{2}\, Q_2 - Q_3 + \frac{1}{2}\, Q_4\right)
\end{aligned}
\right\} \quad (4\text{-}20b)
$$

$$
= \underline{\underline{t_3 \frac{wL^3}{EI_o}}} \qquad (4\text{-}20)
$$

4-6. <u>Load Function - Triangular Load</u>

A tapered beam of I-section, box section or T-section simply supported at the ends, is acted on by a triangular load of maximum intensity q (Figure 4-6)

The bending moment at x, due to triangular load is:

$$
BM_x = \frac{1}{6}\, q(Lx - \frac{x^3}{L}) \qquad (4\text{-}21)
$$

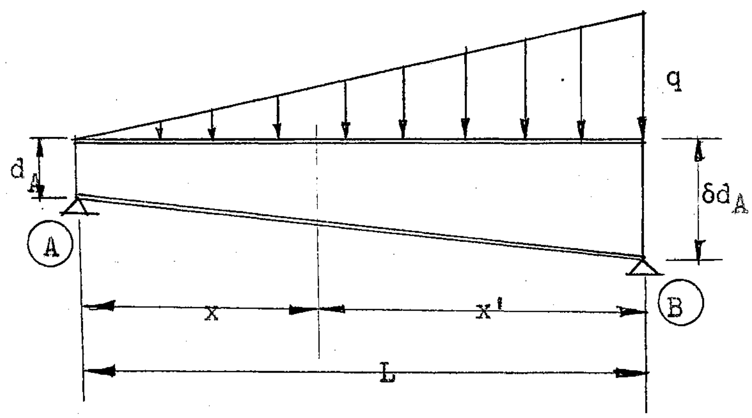


Figure 4-6. A Tapered Beam of I-Section, Box Section or T-Section Triangular Load.

The right end slope (Equation 1-10c)

$$\left.\begin{aligned}
\tau_{BA}^{(TL)} &= \int_A^B \frac{BM_x x dx}{LEI_x} \\
&= \frac{q}{6}\left(\int_o^L \frac{(L^2 - x^2)x^2 dx}{L^2 EI_x}\right) \\
&= \frac{q}{6}\left(\int_o^L \frac{x^2 dx}{EI_x} - \int_o^L \frac{x^4 dx}{L^2 EI_x}\right)
\end{aligned}\right] \quad (4\text{-}22a)$$

or in terms of t, I_o, I_t with notation Q's (Equation 4-7)

$$\left.\begin{aligned}
\tau_{BA}^{(TL)} &= \frac{qL^3}{6\,EI_o}\left(I_o \int_o^1 \frac{t^2 dt}{EI_t} - I_o \int_o^1 \frac{t^4 dt}{EI_t}\right) \\
&= \frac{qL^3}{EI_o}\left(\frac{1}{6} Q_3 - \frac{1}{6} Q_5\right)
\end{aligned}\right] \quad (4\text{-}22b)$$

$$t_6 \frac{qL^3}{EI_o} \tag{4-22}$$

The left end slope (Equation 1-10c) is:

$$\left.\begin{aligned}\tau_{AB}^{(TL)} &= \int_A^B \frac{BM_x\ (L - x)dx}{L\ EI_x} \\ &= \frac{q}{6}\left(\int_A^B \frac{(L^2 - x^2)(L - x)xdx}{L^2 EI_x}\right) \\ &= \frac{q}{6}\left(\int_o^L \frac{Lxdx}{EI_x} - \int_o^L \frac{x^2dx}{EI_x} - \int_o^L \frac{x^3dx}{LEI_x} + \int_o^L \frac{x^4dx}{L^2EI_x}\right)\end{aligned}\right] \tag{4-23a}$$

or in terms of t, I , I with notation Q's (Equation 4-9)

$$\left.\begin{aligned}\tau_{AB}^{(TL)} &= \frac{qL^3}{6\ EI_o}\left(I_o\int_o^1 \frac{tdt}{EI_t} - I_o\int_o^1 \frac{t^2dt}{EI_t} - I_o\int_o^1 \frac{t^3dt}{EI_t}\right. \\ &\qquad \left. + I_o\int_o^1 \frac{t^4dt}{EI_t}\right) \\ &= \frac{qL^3}{EI_o}\left(\frac{1}{6}\ (Q_2 - Q_3 - Q_4 + Q_5)\right)\end{aligned}\right] \tag{4-23b}$$

$$= t_5\ \frac{qL^3}{EI_o} \tag{4-23}$$

4-7. Beam Constants

TABLE A-1
ρ = 2.1

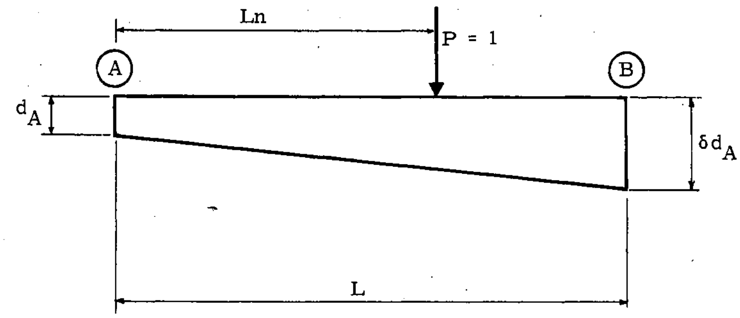
$$F_{AB} = f_1 \frac{L}{EI_A}$$

$$G_{AB} = g \frac{L}{EI_A}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_A}$$

$$\tau_{AB}^{(UL)} = t_3 \frac{wL^3}{EI_A}$$

$$\tau_{AB}^{(DL)} = t_5 \frac{d_A bL^3 q}{EI}$$

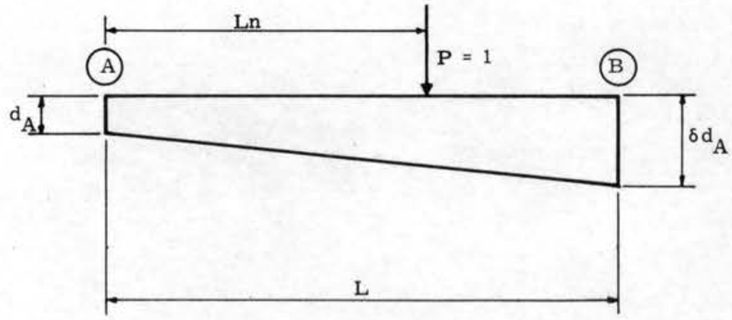


$$\delta = \frac{d_A}{d_B}$$

Coefficients of Angular Functions

δ \ n	Influence Coefficients t_1 .1	.2	.3	.4	.5	.6	.7	.8	.9	f_1	g	t_3	t_5
1.25	.0248	.0410	.0501	.0531	.0511	.0453	.0364	.0255	.0132	.2954	.1312	.0342	.0160
1.50	.0219	.0357	.0430	.0450	.0428	.0376	.0300	.0208	.0106	.2661	.1071	.0290	.0132
1.75	.0197	.0316	.0375	.0388	.0366	.0319	.0253	.0175	.0089	.2427	.0898	.0251	.0113
2.00	.0178	.0282	.0331	.0340	.0319	.0276	.0218	.0150	.0076	.2235	.0767	.0220	.0098
2.25	.0163	.0255	.0296	.0301	.0280	.0241	.0190	.0131	.0066	.2074	.0666	.0195	.0086
2.50	.0150	.0232	.0267	.0269	.0249	.0214	.0168	.0115	.0058	.1937	.0586	.0174	.0077
2.75	.0139	.0212	.0242	.0243	.0224	.0191	.0150	.0102	.0052	.1818	.0521	.0157	.0069
3.00	.0129	.0195	.0221	.0220	.0202	.0172	.0134	.0092	.0047	.1714	.0467	.0143	.0062
3.25	.0120	.0180	.0203	.0201	.0184	.0156	.0122	.0083	.0042	.1622	.0422	.0131	.0057
3.50	.0113	.0167	.0187	.0185	.0168	.0143	.0111	.0076	.0038	.1540	.0383	.0120	.0052
3.75	.0106	.0156	.0173	.0170	.0155	.0131	.0102	.0069	.0035	.1467	.0350	.0111	.0048
4.00	.0100	.0146	.0161	.0158	.0143	.0120	.0093	.0064	.0032	.1400	.0322	.0103	.0044
4.25	.0094	.0137	.0150	.0147	.0132	.0111	.0086	.0059	.0030	.1340	.0297	.0096	.0041
4.50	.0089	.0129	.0140	.0137	.0123	.0104	.0080	.0054	.0027	.1286	.0275	.0090	.0038
4.75	.0085	.0121	.0132	.0128	.0115	.0097	.0075	.0051	.0026	.1235	.0256	.0084	.0036
5.00	.0081	.0115	.0124	.0120	.0108	.0090	.0070	.0047	.0024	.1189	.0239	.0079	.0033
5.25	.0077	.0108	.0117	.0113	.0101	.0085	.0065	.0044	.0022	.1146	.0224	.0074	.0031
5.50	.0074	.0103	.0111	.0106	.0095	.0080	.0061	.0042	.0021	.1110	.0210	.0070	.0030
5.75	.0070	.0098	.0105	.0101	.0090	.0075	.0058	.0039	.0020	.1070	.0197	.0066	.0028
6.00	.0067	.0093	.0099	.0095	.0085	.0071	.0054	.0037	.0019	.1035	.0186	.0063	.0026

TABLE A-2
ρ = 2.1



$$\delta = \frac{d_A}{d_B}$$

$$F_{BA} = f_2 \frac{L}{EI_A}$$

$$G_{BA} = g \frac{L}{EI_A}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_A}$$

$$\tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_A}$$

$$\tau_{BA}^{(\ L)} = t_6 \frac{d_A b L^3 q}{EI_A}$$

Coefficients of Angular Functions

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.25	.0130	.0250	.0351	.0427	.0472	.0476	.0436	.0346	.0202	.2340	.1312	.0314	.0163
1.50	.0106	.0202	.0283	.0341	.0372	.0371	.0336	.0264	.0153	.1730	.1071	.0245	.0128
1.75	.0088	.0168	.0233	.0279	.0301	.0298	.0268	.0208	.0119	.1349	.0898	.0198	.0103
2.00	.0075	.0143	.0196	.0233	.0250	.0250	.0219	.0169	.0096	.1080	.0767	.0164	.0084
2.25	.0065	.0123	.0168	.0198	.0211	.0206	.0182	.0140	.0079	.0885	.0666	.0139	.0071
2.50	.0051	.0107	.0146	.0171	.0181	.0175	.0154	.0118	.0067	.0740	.0586	.0119	.0060
2.75	.0051	.0095	.0128	.0149	.0157	.0151	.0133	.0101	.0057	.0628	.0521	.0103	.0052
3.00	.0045	.0084	.0113	.0131	.0137	.0132	.0115	.0088	.0049	.0540	.0467	.0090	.0045
3.25	.0041	.0075	.0101	.0116	.0122	.0116	.0101	.0077	.0043	.0470	.0422	.0080	.0040
3.50	.0037	.0068	.0091	.0104	.0108	.0103	.0090	.0068	.0038	.0413	.0383	.0071	.0036
3.75	.0034	.0062	.0082	.0094	.0097	.0092	.0080	.0060	.0033	.0365	.0350	.0064	.0032
4.00	.0031	.0056	.0075	.0085	.0088	.0083	.0072	.0054	.0030	.0326	.0322	.0058	.0029
4.25	.0028	.0052	.0068	.0077	.0080	.0075	.0065	.0049	.0027	.0293	.0297	.0053	.0026
4.50	.0026	.0048	.0063	.0071	.0073	.0069	.0059	.0044	.0024	.0264	.0275	.0048	.0024
4.75	.0024	.0044	.0058	.0065	.0067	.0063	.0054	.0040	.0022	.0240	.0256	.0044	.0021
5.00	.0023	.0041	.0053	.0060	.0061	.0057	.0049	.0037	.0020	.0219	.0239	.0040	.0020
5.25	.0021	.0038	.0049	.0055	.0056	.0053	.0045	.0034	.0018	.0200	.0224	.0038	.0018
5.50	.0020	.0035	.0046	.0051	.0052	.0049	.0042	.0031	.0017	.0184	.0210	.0035	.0017
5.75	.0019	.0033	.0043	.0048	.0048	.0045	.0039	.0029	.0016	.0170	.0197	.0032	.0016
6.00	.0018	.0031	.0040	.0045	.0045	.0042	.0036	.0027	.0015	.0157	.0186	.0030	.0017

TABLE A-3
ρ = 2.2

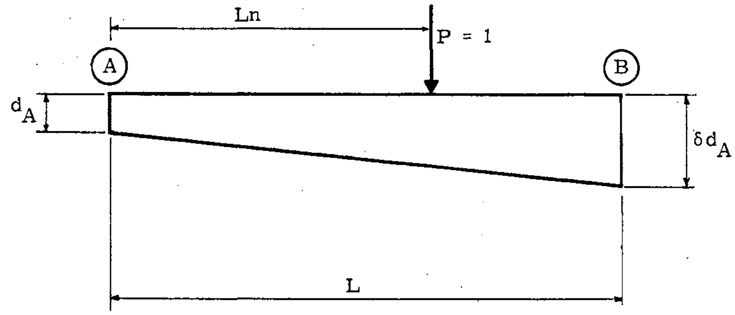
$F_{AB} = f_1 \dfrac{L}{EI_A}$

$G_{AB} = g \dfrac{L}{EI_A}$

$\tau_{AB}^{(LL)} = t_1 \dfrac{L^2}{EI_A}$

$\tau_{AB}^{(UL)} = t_3 \dfrac{wL^3}{EI_A}$

$\tau_{AB}^{(DL)} = t_5 \dfrac{d_A b L^3 q}{EI}$

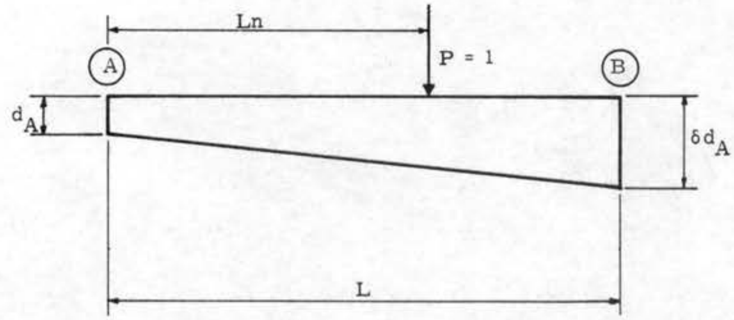


$\delta = \dfrac{d_A}{d_B}$

Coefficients of Angular Functions

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.25	.0247	.0407	.0500	.0525	.0506	.0447	.0360	.0251	.0129	.2940	.1300	.0340	.0160
1.50	.0217	.0353	.0423	.0442	.0421	.0369	.0294	.0204	.0104	.2635	.1050	.0286	.0130
1.75	.0194	.0310	.0367	.0379	.0358	.0311	.0247	.0170	.0087	.2395	.0873	.0245	.0110
2.00	.0175	.0276	.0323	.0330	.0309	.0267	.0210	.0145	.0074	.2200	.0742	.0213	.0095
2.25	.0159	.0248	.0287	.0291	.0271	.0233	.0183	.0126	.0064	.2034	.0640	.0190	.0083
2.50	.0146	.0224	.0257	.0259	.0240	.0205	.0161	.0110	.0056	.1900	.0560	.0168	.0074
2.75	.0134	.0205	.0233	.0233	.0214	.0182	.0143	.0097	.0049	.1774	.0500	.0151	.0066
3.00	.0125	.0188	.0212	.0210	.0193	.0164	.0128	.0087	.0044	.1670	.0442	.0140	.0060
3.25	.0116	.0173	.0194	.0192	.0175	.0148	.0115	.0078	.0040	.1580	.0400	.0125	.0054
3.50	.0108	.0160	.0178	.0175	.0159	.0134	.0104	.0071	.0036	.1500	.0360	.0114	.0050
3.75	.0102	.0149	.0164	.0161	.0146	.0123	.0095	.0065	.0033	.1422	.0330	.0105	.0045
4.00	.0096	.0139	.0152	.0149	.0134	.0113	.0087	.0059	.0030	.1356	.0301	.0097	.0042
4.25	.0090	.0130	.0142	.0138	.0124	.0104	.0081	.0055	.0028	.1300	.0277	.0090	.0038
4.50	.0085	.0122	.0132	.0128	.0115	.0097	.0075	.0051	.0026	.1240	.0256	.0084	.0036
4.75	.0081	.0115	.0124	.0120	.0107	.0090	.0069	.0047	.0024	.1192	.0237	.0079	.0033
5.00	.0077	.0108	.0116	.0112	.0100	.0084	.0065	.0044	.0022	.1150	.0220	.0074	.0031
5.25	.0073	.0102	.0110	.0105	.0094	.0078	.0060	.0041	.0021	.1100	.0206	.0070	.0029
5.50	.0070	.0097	.0103	.0099	.0088	.0073	.0057	.0038	.0019	.1060	.0190	.0065	.0027
5.75	.0067	.0092	.0098	.0093	.0083	.0069	.0053	.0036	.0018	.1030	.0180	.0062	.0026
6.00	.0064	.0087	.0092	.0088	.0078	.0065	.0050	.0034	.0017	.0994	.0170	.0058	.0024

TABLE A-4
$\rho = 2.2$



$$\delta = \frac{d_A}{d_B}$$

$$F_{BA} = f_2 \frac{L}{EI_A}$$

$$G_{BA} = g \frac{L}{EI_A}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_A}$$

$$\tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_A}$$

$$\tau_{BA}^{(\ L)} = t_6 \frac{d_A bL^3 q}{EI_A}$$

Coefficients of Angular Functions

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.25	.0128	.0247	.0348	.0423	.0466	.0470	.0431	.0342	.0200	.2300	.1300	.0308	.0163
1.50	.0103	.0200	.0276	.0333	.0363	.0362	.0328	.0257	.0148	.1690	.1050	.0239	.0125
1.75	.0086	.0163	.0226	.0270	.0291	.0288	.0260	.0201	.0115	.1300	.0873	.0191	.0099
2.00	.0073	.0137	.0189	.0224	.0240	.0235	.0210	.0161	.0092	.1025	.0742	.0157	.0081
2.25	.0063	.0118	.0160	.0189	.0201	.0196	.0173	.0133	.0075	.0833	.0640	.0132	.0067
2.50	.0055	.0102	.0139	.0162	.0171	.0165	.0145	.0111	.0062	.0700	.0560	.0112	.0057
2.75	.0048	.0090	.0121	.0140	.0147	.0142	.0124	.0094	.0053	.0580	.0500	.0097	.0049
3.00	.0043	.0079	.0106	.0123	.0128	.0123	.0107	.0081	.0045	.0497	.0442	.0084	.0042
3.25	.0038	.0071	.0095	.0109	.0113	.0108	.0094	.0071	.0039	.0430	.0400	.0074	.0037
3.50	.0035	.0064	.0085	.0097	.0100	.0095	.0082	.0062	.0034	.0375	.0360	.0066	.0033
3.75	.0032	.0058	.0076	.0087	.0090	.0085	.0073	.0055	.0030	.0330	.0330	.0059	.0029
4.00	.0029	.0052	.0069	.0078	.0080	.0076	.0065	.0049	.0027	.0300	.0300	.0053	.0026
4.25	.0026	.0048	.0063	.0071	.0073	.0068	.0059	.0044	.0024	.0263	.0277	.0048	.0024
4.50	.0024	.0044	.0057	.0065	.0066	.0062	.0053	.0040	.0022	.0240	.0260	.0044	.0021
4.75	.0023	.0040	.0053	.0059	.0060	.0056	.0048	.0036	.0020	.0210	.0237	.0040	.0020
5.00	.0021	.0037	.0049	.0054	.0055	.0052	.0044	.0033	.0018	.0194	.0220	.0037	.0018
5.25	.0020	.0035	.0045	.0050	.0051	.0047	.0040	.0030	.0016	.0180	.0206	.0034	.0016
5.50	.0018	.0032	.0042	.0046	.0047	.0044	.0037	.0027	.0015	.0162	.0190	.0031	.0015
5.75	.0017	.0030	.0039	.0043	.0043	.0040	.0034	.0025	.0014	.0150	.0180	.0029	.0014
6.00	.0016	.0028	.0036	.0040	.0040	.0037	.0032	.0023	.0013	.0137	.0170	.0027	.0013

TABLE A-5
$\rho = 2.3$

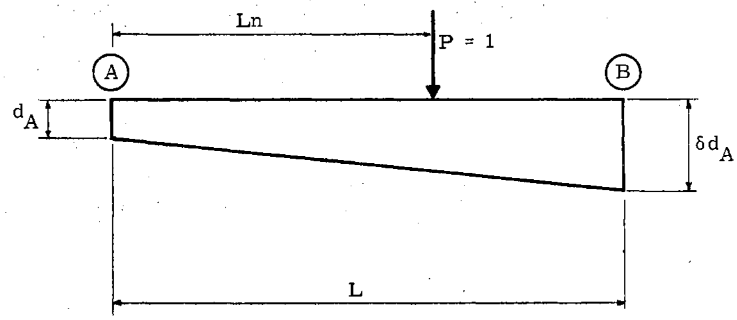
$$F_{AB} = f_1 \frac{L}{EI_A}$$

$$G_{AB} = g \frac{L}{EI_A}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_A}$$

$$\tau_{AB}^{(UL)} = t_3 \frac{wL^3}{EI_A}$$

$$\tau_{AB}^{(DL)} = t_5 \frac{d_A b L^3 q}{EI}$$

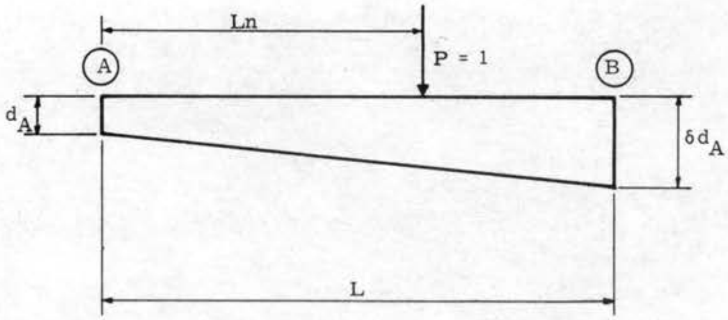


$$\delta = \frac{d_A}{d_B}$$

Coefficients of Angular Functions

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.25	.0245	.0404	.0492	.0520	.0501	.0443	.0356	.0248	.0127	.2921	.1283	.0338	.0153
1.50	.0214	.0348	.0417	.0435	.0414	.0362	.0288	.0200	.0102	.2608	.1028	.0281	.0128
1.75	.0190	.0304	.0360	.0371	.0349	.0303	.0240	.0166	.0084	.2362	.0849	.0240	.0107
2.00	.0171	.0269	.0315	.0321	.0300	.0259	.0204	.0140	.0071	.2161	.0717	.0207	.0092
2.25	.0155	.0241	.0278	.0282	.0261	.0224	.0176	.0121	.0061	.1994	.0615	.0182	.0081
2.50	.0142	.0217	.0249	.0250	.0230	.0197	.0154	.0105	.0053	.1854	.0536	.0161	.0071
2.75	.0130	.0198	.0224	.0223	.0205	.0174	.0136	.0093	.0047	.1732	.0472	.0145	.0063
3.00	.0121	.0181	.0203	.0201	.0184	.0156	.0121	.0083	.0042	.1627	.0419	.0131	.0057
3.25	.0112	.0166	.0185	.0182	.0166	.0140	.0109	.0074	.0037	.1534	.0376	.0119	.0051
3.50	.0104	.0153	.0170	.0166	.0151	.0127	.0098	.0067	.0034	.1452	.0340	.0109	.0046
3.75	.0098	.0142	.0156	.0152	.0138	.0115	.0090	.0061	.0031	.1379	.0308	.0100	.0043
4.00	.0092	.0132	.0144	.0140	.0126	.0106	.0082	.0056	.0028	.1313	.0281	.0092	.0039
4.25	.0086	.0123	.0134	.0130	.0116	.0098	.0075	.0051	.0026	.1253	.0258	.0085	.0036
4.50	.0082	.0115	.0125	.0120	.0108	.0090	.0070	.0047	.0024	.1199	.0238	.0079	.0033
4.75	.0077	.0108	.0117	.0112	.0100	.0084	.0064	.0044	.0022	.1150	.0220	.0074	.0031
5.00	.0073	.0102	.0109	.0105	.0093	.0078	.0060	.0041	.0020	.1105	.0205	.0069	.0029
5.25	.0070	.0096	.0102	.0098	.0087	.0073	.0056	.0038	.0019	.1063	.0191	.0065	.0027
5.50	.0066	.0091	.0097	.0092	.0082	.0068	.0052	.0035	.0018	.1024	.0178	.0061	.0025
5.75	.0063	.0086	.0091	.0087	.0077	.0064	.0049	.0033	.0017	.0988	.0167	.0058	.0024
6.00	.0060	.0082	.0086	.0082	.0072	.0060	.0046	.0031	.0016	.0955	.0157	.0054	.0023

TABLE A-6
$\rho = 2.3$



$$\delta = \frac{d_A}{d_B}$$

$$F_{BA} = f_2 \frac{L}{EI_A}$$

$$G_{BA} = g \frac{L}{EI_A}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_A}$$

$$\tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_A}$$

$$\tau_{BA}^{(\ L)} = t_6 \frac{d_A bL^3 q}{EI_A}$$

Coefficients of Angular Functions

	Influence Coefficients t_2												
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9	f_2	g	t_4	t_6
1.25	.0127	.0244	.0344	.0418	.0460	.0464	.0424	.0337	.0197	.2259	.1283	.0303	.0162
1.50	.0101	.0194	.0270	.0325	.0354	.0353	.0312	.0250	.0144	.1636	.1028	.0233	.0122
1.75	.0083	.0159	.0219	.0261	.0282	.0278	.0248	.0193	.0110	.1240	.0849	.0185	.0096
2.00	.0070	.0133	.0182	.0215	.0230	.0225	.0200	.0154	.0087	.0973	.0717	.0151	.0077
2.25	.0060	.0113	.0154	.0180	.0191	.0185	.0164	.0125	.0071	.0784	.0615	.0126	.0064
2.50	.0052	.0097	.0132	.0153	.0162	.0156	.0137	.0104	.0058	.0645	.0536	.0106	.0054
2.75	.0046	.0085	.0114	.0132	.0139	.0133	.0116	.0088	.0049	.0540	.0472	.0091	.0046
3.00	.0041	.0075	.0100	.0115	.0120	.0115	.0100	.0075	.0042	.0458	.0419	.0079	.0040
3.25	.0036	.0067	.0089	.0101	.0105	.0100	.0086	.0065	.0036	.0394	.0376	.0069	.0034
3.50	.0033	.0060	.0079	.0090	.0093	.0088	.0076	.0057	.0031	.0342	.0340	.0061	.0030
3.75	.0030	.0054	.0071	.0080	.0082	.0078	.0067	.0050	.0028	.0300	.0308	.0054	.0027
4.00	.0027	.0049	.0064	.0072	.0074	.0069	.0059	.0044	.0024	.0265	.0281	.0049	.0024
4.25	.0025	.0044	.0058	.0065	.0066	.0062	.0053	.0040	.0022	.0236	.0258	.0044	.0022
4.50	.0023	.0041	.0053	.0059	.0060	.0056	.0048	.0036	.0019	.0211	.0238	.0040	.0020
4.75	.0021	.0037	.0048	.0054	.0055	.0050	.0043	.0032	.0018	.0190	.0220	.0036	.0018
5.00	.0019	.0034	.0044	.0049	.0050	.0046	.0039	.0029	.0016	.0172	.0205	.0033	.0016
5.25	.0018	.0032	.0041	.0045	.0046	.0042	.0036	.0027	.0015	.0156	.0191	.0030	.0015
5.50	.0017	.0030	.0038	.0042	.0042	.0039	.0033	.0024	.0013	.0143	.0178	.0028	.0014
5.75	.0016	.0027	.0035	.0039	.0039	.0036	.0030	.0022	.0012	.0131	.0167	.0026	.0012
6.00	.0015	.0026	.0033	.0036	.0036	.0033	.0028	.0021	.0011	.0120	.0157	.0024	.0012

TABLE A-7
$\rho = 2.4$

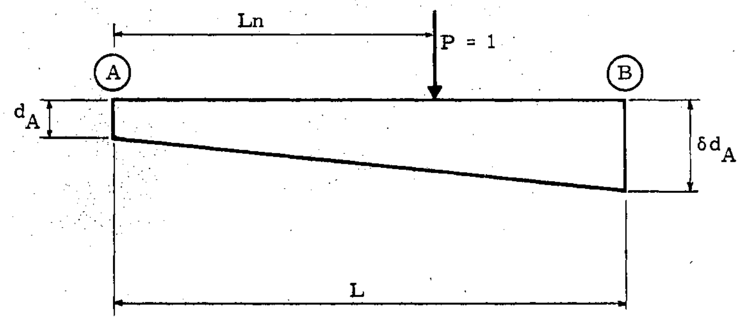
$$F_{AB} = f_1 \frac{L}{EI_A}$$

$$G_{AB} = g \frac{L}{EI_A}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_A}$$

$$\tau_{AB}^{(UL)} = t_3 \frac{wL^3}{EI_A}$$

$$\tau_{AB}^{(DL)} = t_5 \frac{d_A b L^3 q}{EI}$$

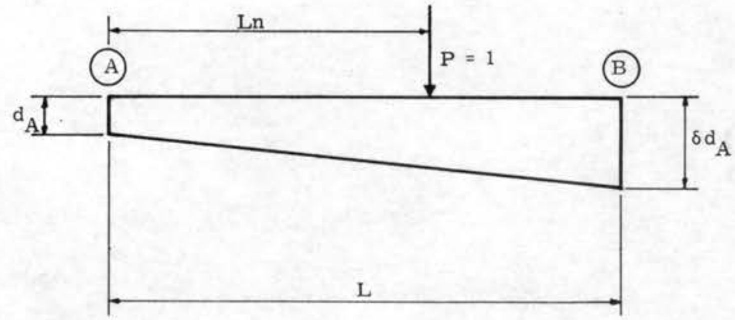


$$\delta = \frac{d_A}{d_B}$$

Coefficients of Angular Functions

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.25	.0243	.0402	.0488	.0516	.0496	.0439	.0352	.0246	.0126	.2905	.1269	.0335	.0153
1.50	.0212	.0343	.0411	.0428	.0407	.0356	.0283	.0196	.0100	.2583	.1008	.0277	.0125
1.75	.0187	.0298	.0352	.0363	.0341	.0295	.0234	.0161	.0082	.2330	.0827	.0234	.0105
2.00	.0168	.0263	.0307	.0312	.0291	.0251	.0198	.0136	.0069	.2126	.0693	.0202	.0090
2.25	.0152	.0235	.0270	.0273	.0252	.0216	.0169	.0116	.0059	.1957	.0592	.0176	.0078
2.50	.0138	.0210	.0240	.0241	.0221	.0189	.0147	.0101	.0051	.1814	.0512	.0156	.0068
2.75	.0127	.0190	.0215	.0214	.0196	.0166	.0130	.0088	.0045	.1692	.0449	.0139	.0060
3.00	.0117	.0174	.0195	.0192	.0175	.0148	.0115	.0078	.0040	.1586	.0398	.0125	.0054
3.25	.0108	.0159	.0177	.0174	.0157	.0133	.0103	.0070	.0035	.1493	.0355	.0113	.0049
3.50	.0101	.0147	.0162	.0158	.0143	.0120	.0093	.0063	.0032	.1411	.0320	.0103	.0044
3.75	.0094	.0136	.0148	.0144	.0130	.0109	.0084	.0057	.0029	.1338	.0289	.0095	.0040
4.00	.0088	.0125	.0137	.0133	.0119	.0100	.0077	.0052	.0026	.1273	.0263	.0087	.0037
4.25	.0083	.0117	.0127	.0122	.0109	.0091	.0070	.0048	.0024	.1214	.0241	.0080	.0034
4.50	.0078	.0109	.0118	.0113	.0101	.0084	.0065	.0044	.0022	.1160	.0222	.0075	.0031
4.75	.0074	.0103	.0109	.0105	.0093	.0080	.0060	.0041	.0020	.1111	.0205	.0069	.0029
5.00	.0070	.0096	.0103	.0098	.0087	.0072	.0056	.0038	.0019	.1066	.0190	.0065	.0027
5.25	.0066	.0091	.0096	.0091	.0081	.0067	.0052	.0035	.0018	.1025	.0176	.0061	.0025
5.50	.0063	.0086	.0090	.0086	.0076	.0063	.0049	.0033	.0016	.0987	.0164	.0057	.0024
5.75	.0060	.0081	.0085	.0080	.0071	.0059	.0045	.0030	.0015	.0951	.0154	.0054	.0022
6.00	.0057	.0077	.0080	.0076	.0067	.0055	.0042	.0029	.0014	.0919	.0144	.0051	.0021

TABLE A-8
$\rho = 2.4$



$$\delta = \frac{d_A}{d_B}$$

$$F_{BA} = f_2 \frac{L}{EI_A}$$

$$G_{BA} = g \frac{L}{EI_A}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_A}$$

$$\tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_A}$$

$$\tau_{BA}^{(\ L)} = t_6 \frac{d_A bL^3 q}{EI_A}$$

Coefficients of Angular Functions

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.25	.0125	.0241	.0339	.0413	.0454	.0458	.0418	.0332	.0193	.2222	.1269	.0300	.0158
1.50	.0099	.0190	.0265	.0318	.0346	.0344	.0311	.0243	.0140	.1587	.1008	.0228	.0118
1.75	.0081	.0154	.0213	.0253	.0272	.0268	.0240	.0186	.0106	.1190	.0827	.0179	.0092
2.00	.0068	.0128	.0175	.0207	.0220	.0215	.0191	.0147	.0083	.0924	.0693	.0145	.0074
2.25	.0058	.0108	.0147	.0172	.0182	.0176	.0155	.0118	.0067	.0738	.0592	.0119	.0061
2.50	.0050	.0093	.0125	.0146	.0153	.0148	.0129	.0098	.0055	.0603	.0512	.0100	.0051
2.75	.0044	.0081	.0108	.0125	.0130	.0125	.0108	.0082	.0046	.0501	.0449	.0086	.0043
3.00	.0038	.0071	.0094	.0108	.0112	.0107	.0093	.0070	.0039	.0423	.0398	.0074	.0037
3.25	.0034	.0063	.0083	.0095	.0098	.0093	.0080	.0060	.0033	.0361	.0355	.0064	.0032
3.50	.0031	.0056	.0074	.0083	.0086	.0081	.0070	.0052	.0029	.0312	.0320	.0057	.0028
3.75	.0028	.0050	.0066	.0074	.0076	.0071	.0061	.0046	.0025	.0272	.0289	.0050	.0025
4.00	.0025	.0045	.0059	.0066	.0068	.0063	.0054	.0040	.0022	.0239	.0263	.0045	.0022
4.25	.0023	.0041	.0053	.0060	.0061	.0057	.0048	.0036	.0020	.0212	.0241	.0040	.0020
4.50	.0021	.0037	.0048	.0054	.0055	.0051	.0043	.0032	.0017	.0189	.0222	.0036	.0018
4.75	.0019	.0034	.0044	.0049	.0050	.0046	.0039	.0029	.0016	.0169	.0205	.0033	.0016
5.00	.0018	.0032	.0040	.0045	.0045	.0042	.0035	.0026	.0014	.0152	.0190	.0030	.0015
5.25	.0017	.0029	.0037	.0041	.0041	.0038	.0032	.0024	.0013	.0138	.0176	.0027	.0013
5.50	.0015	.0027	.0034	.0038	.0038	.0035	.0029	.0022	.0012	.0126	.0164	.0025	.0012
5.75	.0014	.0025	.0032	.0035	.0035	.0032	.0027	.0020	.0011	.0115	.0154	.0023	.0011
6.00	.0013	.0023	.0029	.0032	.0032	.0029	.0025	.0018	.0010	.0105	.0144	.0021	.0010

TABLE A-9
$\rho = 2.5$

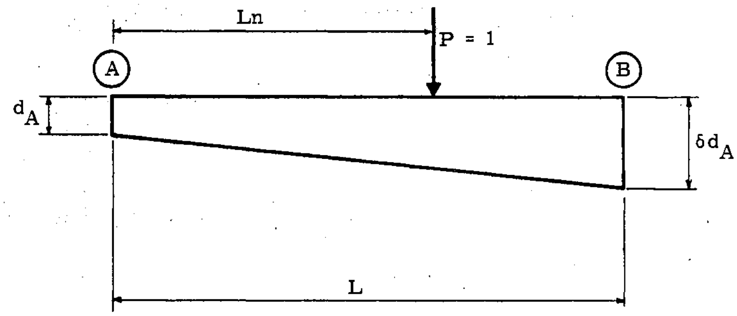
$$F_{AB} = f_1 \frac{L}{EI_A}$$

$$G_{AB} = g \frac{L}{EI_A}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_A}$$

$$\tau_{AB}^{(UL)} = t_3 \frac{wL^3}{EI_A}$$

$$\tau_{AB}^{(DL)} = t_5 \frac{d_A b L^3 q}{EI}$$

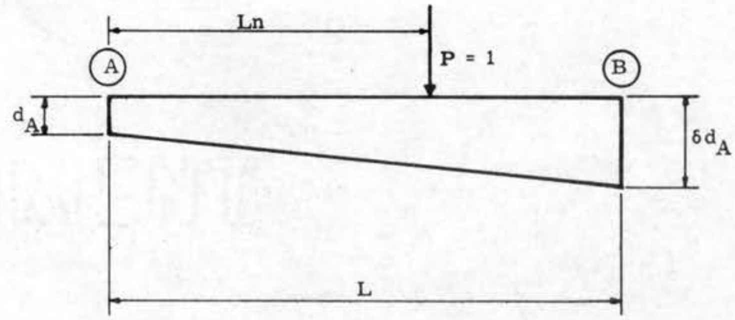


$$\delta = \frac{d_A}{d_B}$$

Coefficients of Angular Functions

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.25	.0242	.0398	.0484	.0512	.0492	.0434	.0349	.0243	.0125	.2889	.1255	.0332	.0151
1.50	.0209	.0339	.0405	.0421	.0400	.0349	.0278	.0192	.0098	.2557	.0989	.0272	.0123
1.75	.0184	.0293	.0345	.0355	.0333	.0288	.0228	.0157	.0080	.2299	.0804	.0229	.0103
2.00	.0164	.0257	.0299	.0304	.0283	.0243	.0191	.0131	.0067	.2091	.0670	.0196	.0087
2.25	.0148	.0228	.0262	.0264	.0244	.0208	.0163	.0112	.0057	.1920	.0568	.0170	.0075
2.50	.0135	.0204	.0232	.0232	.0213	.0181	.0141	.0096	.0049	.1776	.0490	.0150	.0066
2.75	.0123	.0184	.0207	.0206	.0188	.0159	.0124	.0084	.0043	.1653	.0428	.0133	.0058
3.00	.0113	.0168	.0187	.0184	.0167	.0141	.0109	.0074	.0038	.1547	.0377	.0120	.0052
3.25	.0105	.0153	.0169	.0166	.0150	.0126	.0098	.0066	.0033	.1454	.0336	.0108	.0046
3.50	.0097	.0141	.0154	.0150	.0135	.0113	.0088	.0060	.0030	.1372	.0301	.0098	.0042
3.75	.0090	.0130	.0141	.0137	.0123	.0103	.0079	.0054	.0027	.1300	.0272	.0090	.0038
4.00	.0085	.0120	.0130	.0125	.0112	.0094	.0072	.0049	.0025	.1235	.0247	.0082	.0035
4.25	.0079	.0111	.0120	.0115	.0103	.0086	.0066	.0045	.0022	.1176	.0225	.0076	.0032
4.50	.0075	.0104	.0111	.0106	.0095	.0079	.0061	.0041	.0021	.1123	.0207	.0070	.0029
4.75	.0070	.0097	.0103	.0098	.0087	.0073	.0056	.0038	.0019	.1074	.0190	.0065	.0027
5.00	.0066	.0091	.0096	.0092	.0081	.0067	.0052	.0035	.0018	.1030	.0176	.0061	.0025
5.25	.0063	.0086	.0090	.0085	.0075	.0063	.0048	.0032	.0016	.0989	.0163	.0057	.0024
5.50	.0060	.0081	.0085	.0080	.0070	.0058	.0045	.0030	.0015	.0952	.0152	.0053	.0022
5.75	.0057	.0076	.0079	.0075	.0066	.0055	.0042	.0028	.0014	.0917	.0142	.0050	.0021
6.00	.0054	.0072	.0075	.0070	.0062	.0051	.0039	.0026	.0013	.0885	.0133	.0047	.0019

TABLE A-10
$\rho = 2.5$



$$\delta = \frac{d_A}{d_B}$$

$$F_{BA} = f_2 \frac{L}{EI_A}$$

$$G_{BA} = g \frac{L}{EI_A}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_A}$$

$$\tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_A}$$

$$\tau_{BA}^{(\ L)} = t_6 \frac{d_A bL^3 q}{EI_A}$$

Coefficients of Angular Functions

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.25	.0124	.0238	.0335	.0407	.0448	.0451	.0412	.0327	.0190	.2185	.1225	.0296	.0157
1.50	.0097	.0186	.0259	.0311	.0337	.0335	.0303	.0237	.0136	.1540	.0989	.0222	.0116
1.75	.0079	.0150	.0206	.0245	.0263	.0259	.0231	.0179	.0102	.1142	.0804	.0173	.0089
2.00	.0066	.0123	.0169	.0199	.0211	.0206	.0182	.0140	.0079	.0878	.0670	.0139	.0071
2.25	.0055	.0104	.0141	.0164	.0173	.0168	.0147	.0112	.0063	.0695	.0568	.0114	.0058
2.50	.0048	.0089	.0119	.0138	.0145	.0139	.0121	.0092	.0051	.0563	.0490	.0095	.0048
2.75	.0041	.0077	.0102	.0118	.0122	.0117	.0101	.0076	.0042	.0465	.0428	.0081	.0040
3.00	.0036	.0067	.0089	.0101	.0105	.0100	.0086	.0065	.0036	.0390	.0377	.0069	.0034
3.25	.0032	.0059	.0078	.0088	.0091	.0086	.0074	.0055	.0030	.0331	.0336	.0060	.0030
3.50	.0029	.0052	.0069	.0078	.0079	.0075	.0064	.0048	.0026	.0285	.0301	.0052	.0026
3.75	.0026	.0047	.0061	.0069	.0070	.0066	.0056	.0042	.0023	.0247	.0272	.0046	.0023
4.00	.0023	.0042	.0055	.0061	.0062	.0058	.0049	.0037	.0020	.0216	.0247	.0041	.0020
4.25	.0021	.0038	.0049	.0055	.0056	.0052	.0044	.0032	.0018	.0191	.0225	.0037	.0018
4.50	.0020	.0035	.0045	.0049	.0050	.0046	.0039	.0029	.0016	.0169	.0207	.0033	.0016
4.75	.0018	.0032	.0040	.0045	.0045	.0042	.0035	.0026	.0014	.0151	.0190	.0030	.0014
5.00	.0017	.0029	.0037	.0041	.0041	.0038	.0032	.0023	.0013	.0136	.0176	.0027	.0013
5.25	.0015	.0027	.0034	.0037	.0037	.0034	.0029	.0021	.0011	.0122	.0163	.0025	.0012
5.50	.0014	.0025	.0031	.0034	.0034	.0031	.0026	.0019	.0010	.0111	.0152	.0023	.0011
5.75	.0013	.0023	.0029	.0031	.0031	.0029	.0024	.0017	.0009	.0101	.0142	.0021	.0010
6.00	.0012	.0021	.0027	.0029	.0029	.0026	.0016	.0016	.0009	.0092	.0133	.0019	.0009

TABLE A-11
ρ = 2.6

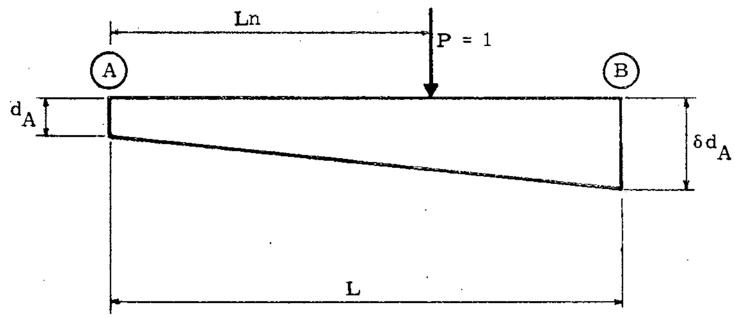
$$F_{AB} = f_1 \frac{L}{EI_A}$$

$$G_{AB} = g \frac{L}{EI_A}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_A}$$

$$\tau_{AB}^{(UL)} = t_3 \frac{wL^3}{EI_A}$$

$$\tau_{AB}^{(DL)} = t_5 \frac{d_A b L^3 q}{EI}$$

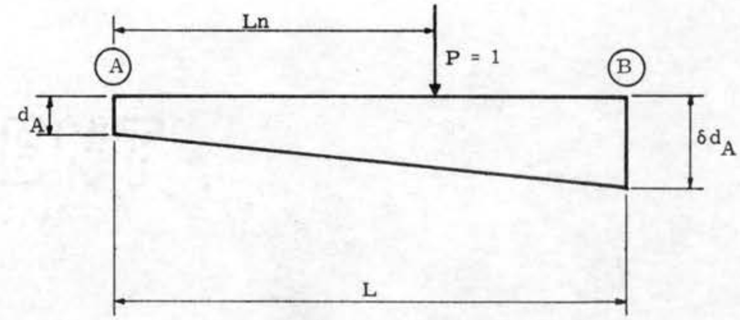


$$\delta = \frac{d_A}{d_B}$$

Coefficients of Angular Functions

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.25	.0240	.0400	.0480	.0507	.0487	.0430	.0345	.0240	.0123	.2873	.1242	.0330	.0148
1.50	.0207	.0334	.0399	.0415	.0393	.0343	.0273	.0189	.0096	.2532	.0969	.0268	.0121
1.75	.0181	.0288	.0338	.0347	.0325	.0281	.0222	.0153	.0078	.2269	.0783	.0224	.0100
2.00	.0161	.0251	.0291	.0296	.0274	.0236	.0185	.0127	.0065	.2058	.0648	.0191	.0085
2.25	.0145	.0222	.0254	.0255	.0235	.0201	.0157	.0108	.0055	.1884	.0547	.0165	.0073
2.50	.0131	.0198	.0224	.0233	.0205	.0174	.0135	.0092	.0047	.1739	.0470	.0145	.0063
2.75	.0119	.0178	.0200	.0197	.0180	.0152	.0118	.0080	.0041	.1616	.0408	.0128	.0055
3.00	.0110	.0161	.0179	.0176	.0159	.0134	.0104	.0071	.0036	.1509	.0359	.0115	.0049
3.25	.0101	.0147	.0162	.0158	.0142	.0120	.0093	.0063	.0032	.1417	.0318	.0103	.0044
3.50	.0094	.0135	.0147	.0143	.0128	.0107	.0083	.0056	.0028	.1335	.0284	.0093	.0040
3.75	.0087	.0124	.0134	.0130	.0116	.0097	.0075	.0051	.0026	.1262	.0256	.0085	.0036
4.00	.0081	.0114	.0123	.0118	.0106	.0088	.0068	.0046	.0023	.1198	.0232	.0078	.0033
4.25	.0076	.0106	.0113	.0109	.0097	.0080	.0062	.0042	.0021	.1140	.0211	.0072	.0030
4.50	.0071	.0099	.0105	.0100	.0089	.0074	.0057	.0038	.0019	.1087	.0193	.0066	.0028
4.75	.0067	.0092	.0097	.0092	.0082	.0068	.0052	.0035	.0018	.1040	.0177	.0061	.0026
5.00	.0063	.0086	.0091	.0086	.0076	.0063	.0048	.0032	.0016	.0996	.0164	.0057	.0024
5.25	.0060	.0081	.0085	.0080	.0070	.0058	.0045	.0030	.0015	.0956	.0151	.0053	.0022
5.50	.0057	.0076	.0079	.0074	.0066	.0054	.0041	.0028	.0014	.0919	.0141	.0050	.0020
5.75	.0054	.0072	.0074	.0070	.0061	.0051	.0039	.0026	.0013	.0885	.0131	.0047	.0019
6.00	.0051	.0068	.0070	.0065	.0057	.0047	.0036	.0024	.0012	.0853	.0122	.0044	.0018

TABLE A-12
$\rho = 2.6$



$$\delta = \frac{d_A}{d_B}$$

$$F_{BA} = f_2 \frac{L}{EI_A}$$

$$G_{BA} = g \frac{L}{EI_A}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_A}$$

$$\tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_A}$$

$$\tau_{BA}^{(\ L)} = t_6 \frac{d_A bL^3 q}{EI_A}$$

Coefficients of Angular Functions

	Influence Coefficients t_2									f_2	g	t_4	t_6
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.25	.0122	.0236	.0331	.0402	.0442	.0445	.0407	.0322	.0188	.2150	.1242	.0291	.0156
1.50	.0095	.0182	.0253	.0304	.0329	.0327	.0295	.0230	.0132	.1494	.0969	.0217	.0113
1.75	.0077	.0145	.0200	.0238	.0255	.0250	.0222	.0172	.0098	.1095	.0783	.0167	.0086
2.00	.0063	.0119	.0163	.0191	0203	.0197	.0174	.0133	.0075	.0834	.0648	.0133	.0068
2.25	.0053	.0100	.0134	.0157	.0165	.0159	.0139	.0106	.0059	.0655	.0547	.0108	.0055
2.50	.0046	.0084	.0113	.0131	.0137	.0131	.0114	.0086	.0048	.0527	.0470	.0090	.0045
2.75	.0039	.0073	.0097	.0111	.0115	.0110	.0095	.0071	.0039	.0432	.0408	.0076	.0038
3.00	.0035	.0063	.0084	.0095	.0098	.0093	.0080	.0060	.0033	.0360	.0359	.0065	.0032
3.25	.0031	.0055	.0073	.0083	.0085	.0080	.0068	.0051	.0028	.0304	.0318	.0056	.0028
3.50	.0027	.0049	.0064	.0072	.0074	.0069	.0059	.0044	.0024	.0260	.0284	.0049	.0024
3.75	.0024	.0044	.0057	.0064	.0065	.0060	.0051	.0038	.0021	.0224	.0256	.0043	.0021
4.00	.0022	.0039	.0051	.0056	.0057	.0053	.0045	.0033	.0018	.0195	.0232	.0038	.0018
4.25	.0020	.0035	.0045	.0050	.0051	.0047	.0040	.0029	.0016	.0171	.0211	.0034	.0016
4.50	.0018	.0032	.0041	.0045	.0045	.0042	.0035	.0026	.0014	.0152	.0193	.0030	.0015
4.75	.0017	.0029	.0037	.0041	.0041	.0038	.0032	.0023	.0013	.0135	.0177	.0027	.0013
5.00	.0015	.0027	.0034	.0037	.0037	.0034	.0029	.0021	.0011	.0121	.0164	.0025	.0012
5.25	.0014	.0024	.0031	.0034	.0034	.0031	.0026	.0019	.0010	.0109	.0151	.0022	.0011
5.50	.0013	.0023	.0028	.0031	.0031	.0028	.0023	.0017	.0009	.0098	.0141	.0020	.0010
5.75	.0012	.0021	.0026	.0028	.0028	.0026	.0021	.0015	.0008	.0089	.0131	.0019	.0009
6.00	.0011	.0019	.0024	.0026	.0026	.0023	.0019	.0014	.0008	.0081	.0122	.0017	.0008

TABLE A-13
$\rho = 2.7$

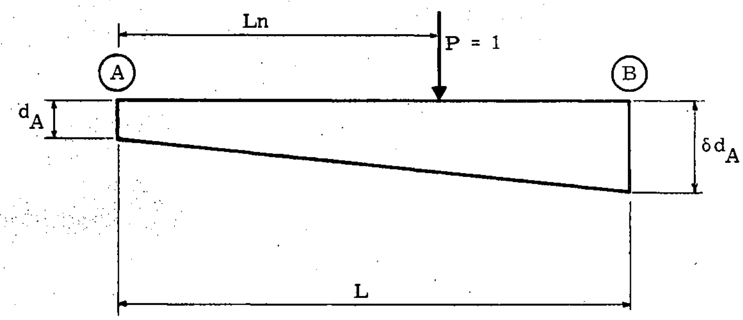
$$F_{AB} = f_1 \frac{L}{EI_A}$$

$$G_{AB} = g \frac{L}{EI_A}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_A}$$

$$\tau_{AB}^{(UL)} = t_3 \frac{wL^3}{EI_A}$$

$$\tau_{AB}^{(DL)} = t_5 \frac{d_A b L^3 q}{EI}$$

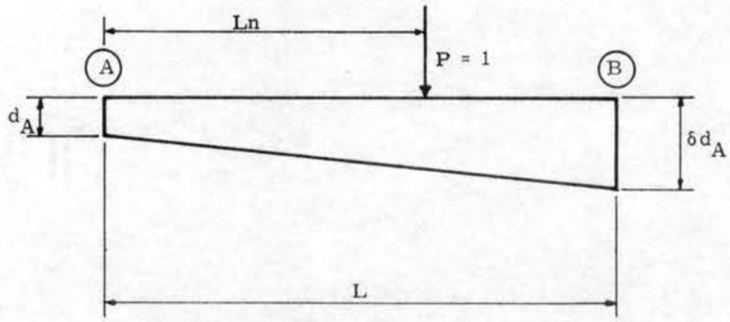


$$\delta = \frac{d_A}{d_B}$$

Coefficients of Angular Functions

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.25	.0239	.0393	.0477	.0503	.0482	.0426	.0342	.0238	.0122	.2858	.1228	.0327	.0146
1.50	.0205	.0330	.0393	.0408	.0386	.0336	.0268	.0185	.0094	.2508	.0950	.0264	.0119
1.75	.0179	.0283	.0331	.0339	.0318	.0274	.0217	.0149	.0076	.2239	.0762	.0219	.0098
2.00	.0158	.0246	.0284	.0287	.0267	.0229	.0179	.0123	.0062	.2025	0627	.0186	.0082
2.25	.0141	.0216	.0247	.0247	.0227	.0194	.0152	.0104	.0052	.1850	.0527	.0160	.0070
2.50	.0128	.0192	.0217	.0215	.0197	.0167	.0130	.0089	.0045	.1704	.0450	.0140	.0061
2.75	.0116	.0172	.0192	.0190	.0172	.0145	.0113	.0077	.0039	.1580	.0389	.0123	.0053
3.00	.0106	.0156	.0172	.0168	.0152	.0128	.0099	.0067	.0034	.1473	.0341	.0110	.0047
3.25	.0098	.0141	.0155	.0151	.0135	.0113	.0088	.0060	.0030	.1380	.0301	.0098	.0042
3.50	.0090	.0129	.0140	.0136	.0122	.0102	.0078	.0053	.0027	.1300	.0268	.0089	.0038
3.75	.0084	.0119	.0128	.0123	.0110	.0092	.0070	.0048	.0024	.1228	.0241	.0081	.0034
4.00	.0078	.0110	.0117	.0112	.0100	.0083	.0064	.0043	.0022	.1163	.0217	.0074	.0031
4.25	.0073	.0101	.0108	.0102	.0091	.0076	.0058	.0039	.0020	.1106	.0197	.0068	.0028
4.50	.0068	.0094	.0099	.0094	.0083	.0069	.0053	.0036	.0018	.1054	.0180	.0062	.0026
4.75	.0064	.0087	.0092	.0087	.0077	.0063	.0049	.0033	.0017	.1007	.0165	.0058	.0024
5.00	.0061	.0082	.0085	.0080	.0071	.0059	.0045	.0030	.0015	.0963	.0152	.0054	.0022
5.25	.0057	.0076	.0080	.0075	.0066	.0054	.0041	.0028	.0014	.0924	.0141	.0050	.0021
5.50	.0054	.0072	.0074	.0070	.0061	.0050	.0038	.0026	.0013	.0888	.0130	.0047	.0019
5.75	.0051	.0067	.0070	.0065	.0057	.0047	.0036	.0024	.0012	.0854	.0121	.0044	.0018
6.00	.0049	.0064	.0065	.0061	.0053	.0044	.0033	.0022	.0011	.0823	.0113	.0041	.0017

TABLE A-14
ρ = 2.7



$$\delta = \frac{d_A}{d_B}$$

$$F_{BA} = f_2 \frac{L}{EI_A}$$

$$G_{BA} = g \frac{L}{EI_A}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_A}$$

$$\tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_A}$$

$$\tau_{BA}^{(\ L)} = t_6 \frac{d_A bL^3 q}{EI_A}$$

Coefficients of Angular Functions

	Influence Coefficients t_2												
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9	f_2	g	t_4	t_6
1.25	.0121	.0233	.0327	.0397	.0436	.0439	.0400	.0317	.0184	.2114	.1228	.0287	.0154
1.50	.0093	.0178	.0248	.0297	.0322	.0319	.0287	.0224	.0128	.1450	.0950	.0212	.0110
1.75	.0075	.0141	.0194	.0230	.0247	.0242	.0215	.0166	.0094	.1051	.0762	.0162	.0083
2.00	.0061	.0115	.0157	.0184	.0195	.0189	.0166	.0127	.0072	.0793	.0627	.0128	.0065
2.25	.0051	.0096	.0129	.0150	.0157	.0151	.0132	.0100	.0056	.0617	.0527	.0103	.0052
2.50	.0044	.0081	.0108	.0124	.0130	.0124	.0107	.0081	.0045	.0493	.0450	.0085	.0042
2.75	.0038	.0069	.0092	.0105	.0108	.0103	.0089	.0067	.0037	.0401	.0389	.0071	.0035
3.00	.0033	.0060	.0079	.0089	.0092	.0087	.0074	.0056	.0031	.0332	.0341	.0061	.0030
3.25	.0029	.0052	.0068	.0077	.0079	.0074	.0063	.0047	.0026	.0280	.0301	.0052	.0026
3.50	.0026	.0046	.0060	.0067	.0068	.0064	.0054	.0040	.0022	.0237	.0268	.0045	.0022
3.75	.0023	.0041	.0053	.0059	.0060	.0056	.0047	.0035	.0019	.0204	.0241	.0040	.0019
4.00	.0021	.0037	.0047	.0052	.0053	.0049	.0041	.0030	.0016	.0177	.0217	.0035	.0017
4.25	.0019	.0033	.0042	.0046	.0047	.0043	.0036	.0027	.0014	.0154	.0197	.0031	.0015
4.50	.0017	.0030	.0038	.0042	.0041	.0038	.0032	.0023	.0013	.0136	.0180	.0028	.0013
4.75	.0015	.0027	.0034	.0037	.0037	.0034	.0029	.0021	.0011	.0121	.0165	.0025	.0012
5.00	.0014	.0025	.0031	.0034	.0033	.0031	.0026	.0019	.0010	.0107	.0152	.0022	.0011
5.25	.0013	.0022	.0028	.0031	.0030	.0028	.0023	.0017	.0009	.0096	.0141	.0020	.0010
5.50	.0012	.0021	.0026	.0028	.0028	.0025	.0021	.0015	.0008	.0087	.0130	.0019	.0009
5.75	.0011	.0019	.0024	.0026	.0025	.0023	.0019	.0014	.0007	.0078	.0121	.0017	.0008
6.00	.0010	.0018	.0022	.0023	.0023	.0021	.0017	.0013	.0007	.0071	.0113	.0015	.0007

TABLE A-15
$\rho = 2.8$

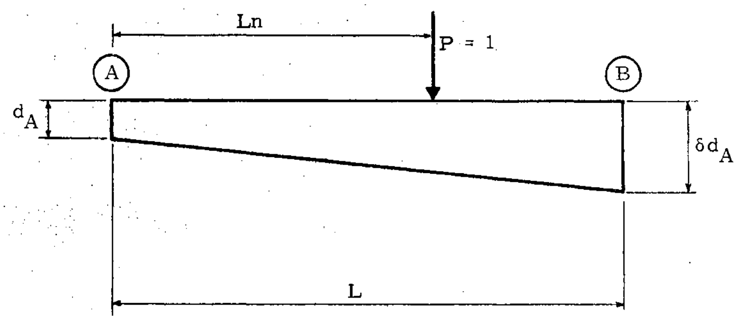
$$F_{AB} = f_1 \frac{L}{EI_A}$$

$$G_{AB} = g \frac{L}{EI_A}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_A}$$

$$\tau_{AB}^{(UL)} = t_3 \frac{wL^3}{EI_A}$$

$$\tau_{AB}^{(DL)} = t_5 \frac{d_A b L^3 q}{EI}$$

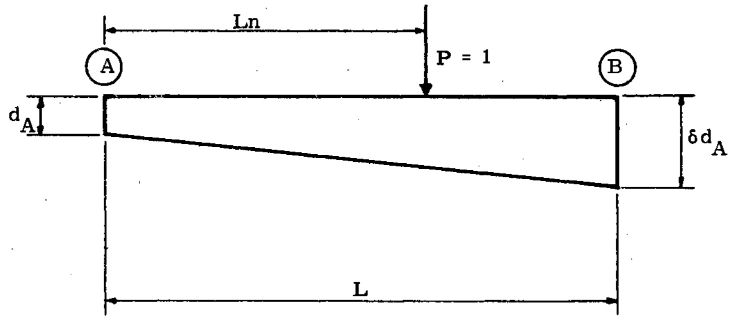


$$\delta = \frac{d_A}{d_B}$$

Coefficients of Angular Functions

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.25	.0237	.0390	.0473	.0500	.0478	.0422	.0338	.0235	.0120	.2842	.1215	.0324	.0145
1.50	.0202	.0326	.0387	.0402	.0380	.0331	.0263	.0182	.0093	.2484	.0932	.0259	.0117
1.75	.0176	.0278	.0325	.0332	.0310	.0268	.0211	.0145	.0074	.2210	.0742	.0214	.0096
2.00	.0155	.0240	.0277	.0280	.0259	.0222	.0174	.0119	.0060	.1993	.0607	.0181	.0080
2.25	.0138	.0211	.0240	.0240	.0220	.0187	.0146	.0100	.0051	.1816	.0507	.0155	.0068
2.50	.0124	.0187	.0210	.0208	.0189	.0160	.0125	.0085	.0043	.1669	0431	.0135	.0058
2.75	.0113	.0167	.0185	.0182	.0165	.0139	.0108	.0073	.0037	.1545	.0371	.0119	.0051
3.00	.0103	.0150	.0165	.0161	.0145	.0122	.0094	.0064	.0032	.1438	.0324	.0105	.0045
3.25	.0095	.0136	.0148	.0144	.0129	.0108	.0083	.0056	.0028	.1346	.0285	.0094	.0040
3.50	.0087	.0124	.0134	.0129	.0115	.0096	.0074	.0050	.0025	.1265	.0253	.0085	.0036
3.75	.0081	.0113	.0122	.0117	.0104	.0087	.0067	.0045	.0023	.1194	.0227	.0077	.0032
4.00	.0075	.0104	.0111	.0106	.0094	.0078	.0060	.0041	.0020	.1130	.0204	.0070	.0029
4.25	.0070	.0096	.0102	.0097	.0086	.0071	.0054	.0037	.0018	.1073	.0185	.0064	.0027
4.50	.0066	.0089	.0094	.0089	.0078	.0065	.0050	.0033	.0017	.1022	.0169	.0059	.0024
4.75	.0062	.0083	.0087	.0082	.0072	.0059	.0045	.0031	.0015	.0975	.0154	.0054	.0022
5.00	.0058	.0077	.0080	.0075	.0066	.0055	.0042	.0028	.0014	.0933	.0142	.0051	.0021
5.25	.0055	.0072	.0075	.0070	.0061	.0051	.0039	.0026	.0013	.0894	.0131	.0047	.0019
5.50	.0052	.0068	.0070	.0065	.0057	.0047	.0036	.0024	.0012	.0858	.0121	.0044	.0018
5.75	.0049	.0064	.0065	.0061	.0053	.0044	.0033	.0022	.0011	.0825	.0112	.0041	.0017
6.00	.0046	.0060	.0061	.0056	.0049	.0041	.0031	.0021	.0010	.0795	.0105	.0038	.0016

TABLE A-16
$\rho = 2.8$



$$\delta = \frac{d_A}{d_B}$$

$$F_{BA} = f_2 \frac{L}{EI_A}$$

$$G_{BA} = g \frac{L}{EI_A}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_A}$$

$$\tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_A}$$

$$\tau_{BA}^{(\ L)} = t_6 \frac{d_A b L^3 q}{EI_A}$$

Coefficients of Angular Functions

	Influence Coefficients t_2												
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9	f_2	g	t_4	t_6
1.25	.0120	.0230	.0324	.0392	.0431	.0433	.0395	.0312	.0181	.2079	.1215	.0283	.0152
1.50	.0092	.0175	.0243	.0290	.0314	.0311	.0280	.0218	.0125	.1408	.0932	.0201	.0107
1.75	.0073	.0136	.0189	.0223	.0239	.0233	.0207	.0160	.0090	.1009	.0742	.0157	.0080
2.00	.0059	.0111	.0151	.0177	.0187	.0181	.0159	.0121	.0068	.0754	.0607	.0123	.0062
2.25	.0049	.0092	.0124	.0143	.0150	.0144	.0125	.0095	.0053	.0582	.0507	.0098	.0050
2.50	.0042	.0077	.0103	.0118	.0123	.0117	.0102	.0076	.0042	.0461	.0431	.0081	.0040
2.75	.0036	.0066	.0087	.0099	.0102	.0097	.0083	.0062	.0034	.0373	.0371	.0067	.0033
3.00	.0031	.0057	.0074	.0084	.0086	.0081	.0069	.0052	.0028	.0307	.0324	.0057	.0028
3.25	.0027	.0049	.0064	.0072	.0074	.0069	.0059	.0043	.0024	.0256	.0285	.0049	.0024
3.50	.0024	.0043	.0056	.0063	.0063	.0059	.0050	.0037	.0020	.0217	.0253	.0042	.0020
3.75	.0022	.0038	.0049	.0055	.0055	.0051	.0043	.0032	.0017	.0185	.0227	.0037	.0018
4.00	.0019	.0034	.0044	.0048	.0048	.0045	.0038	.0028	.0015	.0160	.0204	.0032	.0015
4.25	.0017	.0031	.0039	.0043	.0043	.0039	.0033	.0024	.0013	.0140	.0185	.0028	.0014
4.50	.0016	.0028	.0035	.0038	.0038	.0035	.0029	.0021	.0011	.0122	.0169	.0025	.0012
4.75	.0014	.0025	.0031	.0034	.0034	.0031	.0026	.0019	.0010	.0110	.0154	.0023	.0011
5.00	.0013	.0023	.0028	.0031	.0030	.0028	.0023	.0017	.0009	.0096	.0142	.0020	.0010
5.25	.0012	.0021	.0026	.0028	.0027	.0025	.0021	.0015	.0008	.0086	.0131	.0018	.0009
5.50	.0011	.0019	.0023	.0025	.0025	.0023	.0019	.0014	.0007	.0077	.0121	.0017	.0008
5.75	.0010	.0017	.0022	.0023	.0023	.0020	.0017	.0012	.0007	.0070	.0112	.0015	.0007
6.00	.0010	.0016	.0020	.0021	.0021	.0019	.0015	.0011	.0006	.0063	.0105	.0014	.0007

TABLE A-17
ρ = 2.9

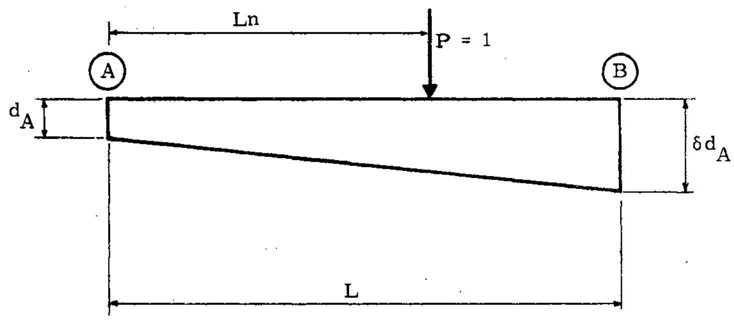
$$F_{AB} = f_1 \frac{L}{EI_A}$$

$$G_{AB} = g \frac{L}{EI_A}$$

$$\tau_{AB}^{(LL)} = t_1 \frac{L^2}{EI_A}$$

$$\tau_{AB}^{(UL)} = t_3 \frac{wL^3}{EI_A}$$

$$\tau_{AB}^{(DL)} = t_5 \frac{d_A bL^3 q}{EI}$$

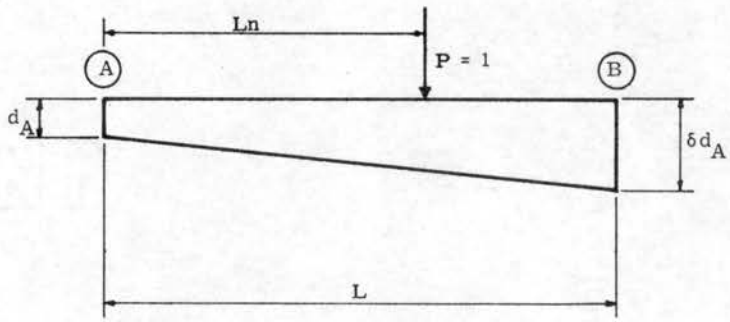


$$\delta = \frac{d_A}{d_B}$$

Coefficients of Angular Functions

	Influence Coefficients t_1									f_1	g	t_3	t_5
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9				
1.25	.0235	.0387	.0469	.0494	.0474	.0417	.0335	.0233	.0119	.2827	.1201	.0323	.0138
1.50	.0200	.0321	.0382	.0395	.0373	.0325	.0258	.0178	.0091	.2460	.0914	.0255	.0115
1.75	.0173	.0273	.0318	.0325	.0303	.0261	.0206	.0141	.0072	.2182	.0722	.0210	.0093
2.00	.0152	.0235	.0270	.0272	.0252	.0215	.0169	.0115	.0058	.1962	.0587	.0176	.0077
2.25	.0135	.0205	.0232	.0232	.0213	.0181	.0141	.0096	.0049	.1784	.0488	.0150	.0066
2.50	.0121	.0181	.0203	.0200	.0182	.0154	.0120	.0081	.0041	.1636	.0413	.0130	.0056
2.75	.0110	.0161	.0179	.0175	.0158	.0133	.0103	.0070	.0035	.1512	.0355	.0114	.0049
3.00	.0100	.0145	.0159	.0154	.0139	.0116	.0090	.0061	.0031	.1405	.0308	.0101	.0043
3.25	.0091	.0131	.0142	.0137	.0123	.0103	.0079	.0054	.0027	.1313	.0271	.0090	.0038
3.50	.0084	.0119	.0128	.0123	.0110	.0091	.0070	.0047	.0024	.1233	.0240	.0081	.0034
3.75	.0078	.0109	.0116	.0111	.0098	.0082	.0063	.0042	.0021	.1162	.0214	.0073	.0031
4.00	.0072	.0100	.0106	.0100	.0089	.0074	.0057	.0038	.0019	.1099	.0192	.0067	.0028
4.25	.0067	.0092	.0097	.0091	.0081	.0067	.0051	.0035	.0017	.1043	.0174	.0061	.0025
4.50	.0063	.0085	.0089	.0084	.0074	.0061	.0047	.0031	.0016	.0992	.0158	.0056	.0023
4.75	.0059	.0079	.0082	.0077	.0067	.0056	.0043	.0029	.0014	.0946	.0144	.0051	.0021
5.00	.0055	.0073	.0076	.0071	.0062	.0051	.0039	.0026	.0013	.0904	.0132	.0048	.0019
5.25	.0052	.0068	.0070	.0066	.0057	.0047	.0036	.0024	.0012	.0866	.0122	.0044	.0018
5.50	.0050	.0064	.0066	.0061	.0053	.0044	.0033	.0022	.0011	.0831	.0112	.0041	.0017
5.75	.0046	.0060	.0061	.0057	.0049	.0041	.0031	.0021	.0010	.0798	.0104	.0038	.0016
6.00	.0044	.0056	.0057	.0053	.0046	.0038	.0029	.0019	.0010	.0768	.0097	.0036	.0014

TABLE A-18
ρ = 2.9



$$\delta = \frac{d_A}{d_B}$$

$$F_{BA} = f_2 \frac{L}{EI_A}$$

$$G_{BA} = g \frac{L}{EI_A}$$

$$\tau_{BA}^{(LL)} = t_2 \frac{L^2}{EI_A}$$

$$\tau_{BA}^{(UL)} = t_4 \frac{wL^3}{EI_A}$$

$$\tau_{BA}^{(\ L)} = t_6 \frac{d_A b L^3 q}{EI_A}$$

Coefficients of Angular Functions

	Influence Coefficients t_2												
δ \ n	.1	.2	.3	.4	.5	.6	.7	.8	.9	f_2	g	t_4	t_6
1.25	.0119	.0228	.0319	.0387	.0425	.0427	.0389	.0307	.0179	.2045	.1201	.0277	.0156
1.50	.0090	.0171	.0237	.0284	.0307	.0304	.0272	.0212	.0121	.1366	.0914	.0202	.0105
1.75	.0071	.0134	.0183	.0216	.0231	.0226	.0200	.0154	.0087	.0968	.0722	.0151	.0078
2.00	.0057	.0107	.0146	.0170	.0179	.0173	.0152	.0116	.0065	.0717	.0587	.0118	.0060
2.25	.0047	.0088	.0118	.0137	.0143	.0137	.0119	.0090	.0050	.0549	.0488	.0094	.0047
2.50	.0040	.0074	.0098	.0112	.0116	.0110	.0095	.0072	.0040	.0431	.0413	.0076	.0038
2.75	.0034	.0062	.0082	.0094	.0096	.0091	.0078	.0058	.0032	.0347	.0355	.0063	.0031
3.00	.0030	.0054	.0070	.0079	.0081	.0076	.0065	.0048	.0026	.0284	.0308	.0053	.0026
3.25	.0026	.0046	.0060	.0068	.0069	.0064	.0054	.0040	.0022	.0236	.0271	.0045	.0022
3.50	.0023	.0041	.0052	.0058	.0059	.0055	.0046	.0034	.0018	.0198	.0240	.0039	.0019
3.75	.0020	.0036	.0046	.0051	.0051	.0047	.0040	.0029	.0016	.0169	.0214	.0034	.0016
4.00	.0018	.0032	.0040	.0045	.0044	.0041	.0034	.0025	.0014	.0145	.0192	.0030	.0014
4.25	.0016	.0028	.0036	.0039	.0039	.0036	.0030	.0022	.0012	.0126	.0174	.0026	.0013
4.50	.0015	.0026	.0032	.0035	.0035	.0032	.0026	.0019	.0010	.0110	.0158	.0023	.0011
4.75	.0013	.0023	.0029	.0031	.0031	.0028	.0023	.0017	.0009	.0097	.0144	.0021	.0010
5.00	.0012	.0021	.0026	.0028	.0028	.0025	.0021	.0015	.0008	.0086	.0132	.0019	.0009
5.25	.0011	.0019	.0024	.0025	.0025	.0022	.0019	.0013	.0007	.0076	.0122	.0017	.0008
5.50	.0010	.0017	.0021	.0023	.0022	.0020	.0017	.0012	.0006	.0068	.0112	.0015	.0007
5.75	.0009	.0016	.0020	.0021	.0020	.0018	.0015	.0011	.0006	.0061	.0104	.0014	.0006
6.00	.0009	.0015	.0018	.0019	.0019	.0017	.0014	.0010	.0005	.0055	.0097	.0013	.0006

CHAPTER V

NUMERICAL EXAMPLES

Two examples are presented to demonstrate the application of The String Polygon Method of analysis. The width of the members of the structures analyzed, is taken as a unity. Units for various values are in terms of kips, feet, or kip-feet.

5-1. Example Number 1

A two panel continuous wedged frame with base fixed shown in Figure 5-1 is considered. The forces at the middle hinges (C) and (F) are required.

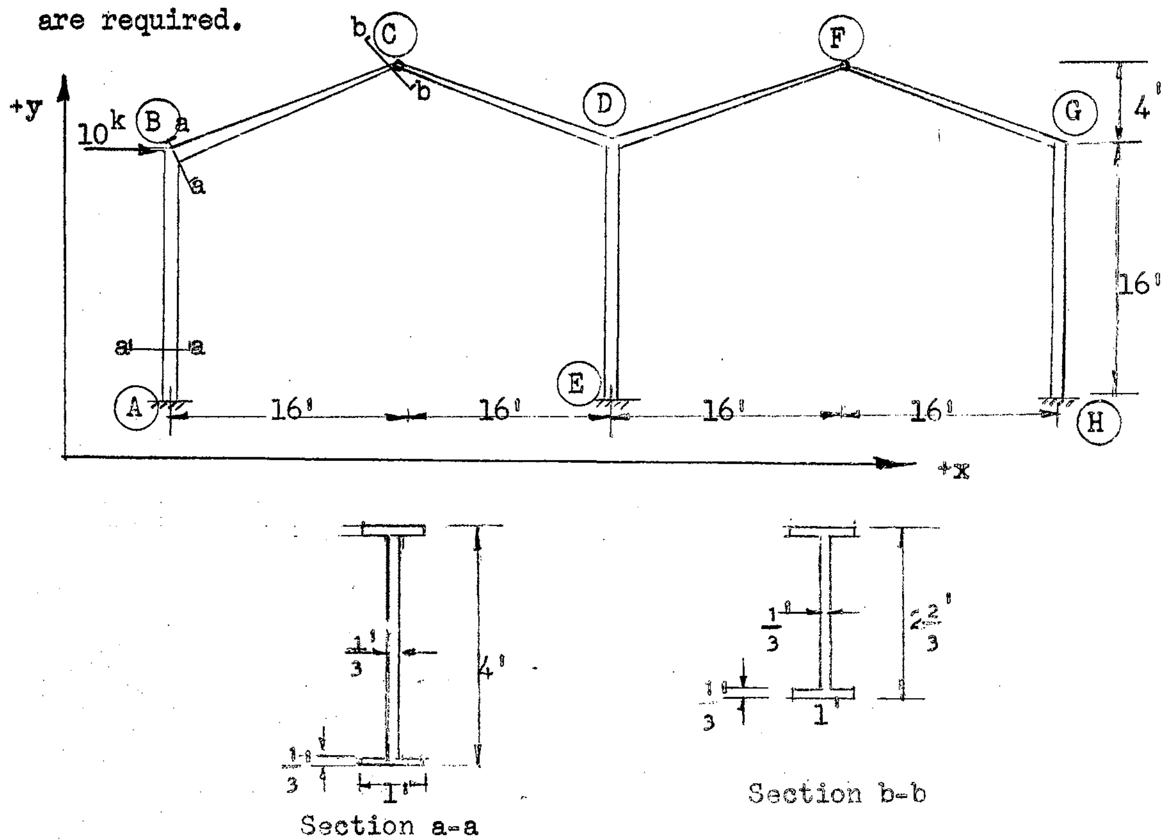


Figure 5-1 Basic Structure

(a) Shape factor, Elastic constants and load functions.

$$I_o = \frac{1}{12}(1)\left(\frac{8}{3}\right)^3 - \frac{2}{12}\left(\frac{1}{3}\right) 2^3 = 1.137 \text{ ft}^3$$

$$I_{col} = \frac{1}{12}(1)\, 4^3 - \frac{2}{12}\left(\frac{1}{3}\right)\left(\frac{3}{10}\right)^3 = 3.27 \text{ ft}^3$$

$$\rho = \frac{\log 3.27/1.137}{\log 1.5} = 2.62$$

TABLE 5-1a

ELASTIC CONSTANTS AND LOAD FUNCTIONS

Member	ρ	Table	h_o ft.	h_o ft.	Flex. Coeff. f_1	Flex. Coeff. f_2	C.O.V. Coeff. (g)
AB	3		4	4	.333	.333	.167
BC	2.6	A-11 A-12	2.66	4	.2533	.1494	.0969

TABLE 5-1b

FINAL ELASTIC CONSTANTS

Member	$\frac{I_{col}}{I_o}$	$F = fL/EI_o$		GEI_o	Angular Load Function	
		$F_{ij}EI_o$	$F_{ji}EI_o$		$\tau_{ij}EI_o$	$\tau_{ji}EI_o$
AB	2.88	1.85	1.85	.925	-	-
BC	1.00	4,18	2.48	1.60	-	-

(b) Joint Moments (Equation 2-1)

$M_B = 4X_1 + 16Y_1$

$M_A = 20X_1 + 16Y_1 - 160$

$M_{DC} = 4X_1 - 16Y_1$

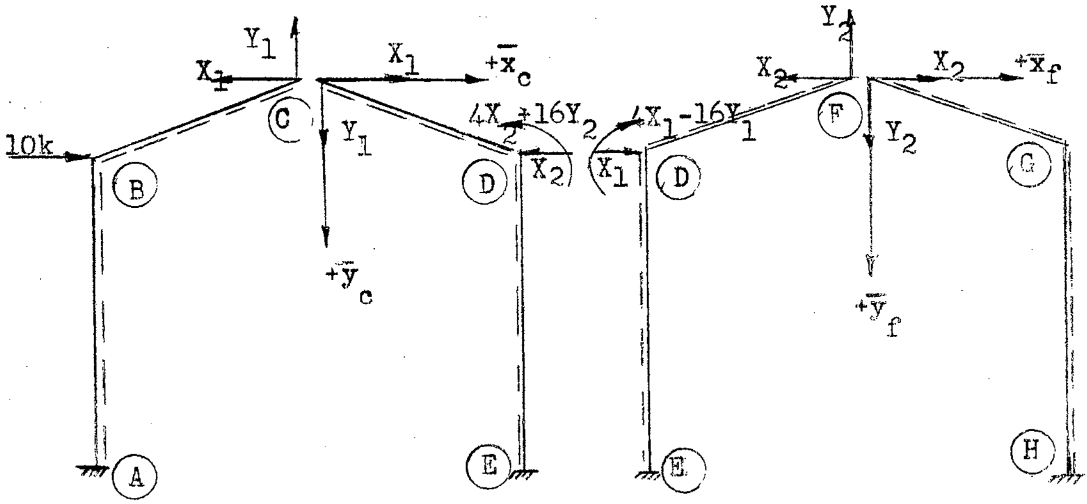


Figure 5-2. Separate Frames

$M_{DE} = 4X_1 - 16Y_1 - 4X_2 - 16Y_2$

$M_E = 20X_1 - 16Y_1 - 20X_2 - 16Y_2$

$M_{DF} = -4X_2 - 16Y_2$

$M_G = 16Y_2 - 4X_2$

$M_H = 16Y_2 - 20X_2$

(c) Joint Elastic weights (Equation 2-2)

$\bar{P}_A = 1.85\ (20X_1 + 16Y_1 - 160) + .925\ (4X_1 + 16Y_1)$

$= 40.7X_1 + 44.4Y_1 - 286$

$\bar{P}_B = 1.85(4X_1 + 16Y_1) + .925(20X_1 + 16Y_1 - 160) + 2.48(4X_1 + 16Y_1)$

$= 35.8X_1 + 84.1Y_1 - 148$

$\bar{P}_D^{(CDE)} = 2.48(4X_1 - 16Y_1) + 1.85(4X_1 - 16Y_1 - 4X_2 - 16Y_2) + .925$

$(20X_1 - 16Y_1 - 20X_2 - 16Y_2)$

$= 35.8X_1 - 84.1Y_1 - 25.9X_2 - 44.4Y_2$

$$\bar{P}_E = 1.85(20X_1 - 16Y_1 - 20X_2 - 16Y_2) + .025(4X_2 - 16Y_2)$$

$$= 40.7X_1 - 44.4Y_1 - 40.7X_2 - 44.4Y_2$$

$$\bar{P}_D^{(EDF)} = 2.48(-4X_2 - 16Y_2) + 1.85(4X_1 - 16Y_1 - 4X_2 - 16Y_2)$$

$$+ .925(20X_1 - 16Y_1 - 20X_2 - 16Y_2)$$

$$= 25.9X_1 - 44.4Y_1 - 35.9X_2 - 84.1Y_2$$

$$\bar{P}_G = 2.48(16Y_2 - 4X_2) + 1.85(16Y_2 - 4X_2) + .925(16Y_2 - 20X_2)$$

$$= 84.1Y_2 - 35.9X_2$$

$$\bar{P}_H = 1.85(16Y_2 - 20X_2) + .925(16Y_2 - 4X_2) = 44.4Y_2 - 40.7X_2$$

(d) Elasto-static equations (Equation 2-3)

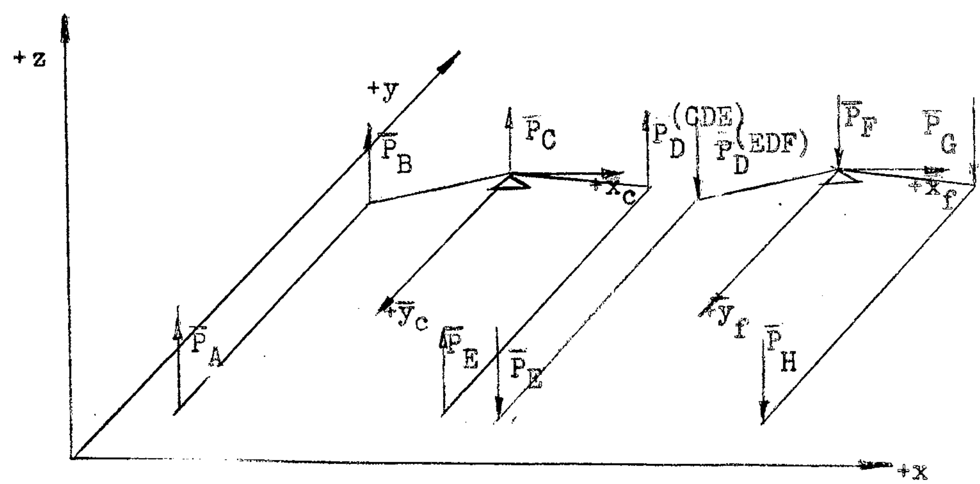


Figure 5-3. Conjugate Structure

$$\sum M_{\bar{x}_c} = 0; \qquad 20(\bar{P}_A + \bar{P}_E) + 4(\bar{P}_B + \bar{P}_D^{(CDE)}) = 0$$

$$478.6X_1 - 229.4X_2 - 255.4Y_2 - 1530 = 0$$

$\sum M_{\bar{y}_c} = 0$

$$16(\bar{P}_A + \bar{P}_B) - 16(\bar{P}_E + \bar{P}_D^{(CDE)}) = 0$$

$$251Y_1 + 66.6X_2 + 88.8Y_2 - 435 = 0$$

$\sum M_{\bar{x}_f} = 0$

$$20(\bar{P}_E + \bar{P}_H) + 4(\bar{P}_G + \bar{P}_D^{(EDF)}) = 0$$

$$229.4X_1 - 266.6Y_1 - 478.6X_2 = 0$$

$\sum M_{\bar{y}_f} = 0$

$$16(\bar{P}_E + \bar{P}_D^{(EDF)}) - 16(\bar{P}_G + \bar{P}_H) = 0$$

$$66.6X_1 - 88.8Y_1 - 251.0Y_2 = 0$$

(e) Force Matrix

TABLE 5-2.

FORCE MATRIX

$$\begin{bmatrix} 478.6 & - & -229.4 & -266.4 \\ - & 251.0 & 66.6 & 88.8 \\ 229.4 & -266.6 & -478.6 & - \\ 66.6 & -88.8 & - & -251.0 \end{bmatrix} \begin{bmatrix} X_1 \\ Y_1 \\ X_2 \\ Y_2 \end{bmatrix} = \begin{bmatrix} 1530 \\ 435 \\ 0 \\ 0 \end{bmatrix}$$

$X_1 = 4.34$ kips

$Y_1 = 1.052$ kips

$X_2 = 1.49$ kips

$Y_2 = 0.775$ kips

5-2. Example Number 2

A two hinge frame with vertical and horizontal loads is shown in Figure 5-1 and Figure 5-2. The horizontal reactions are required.

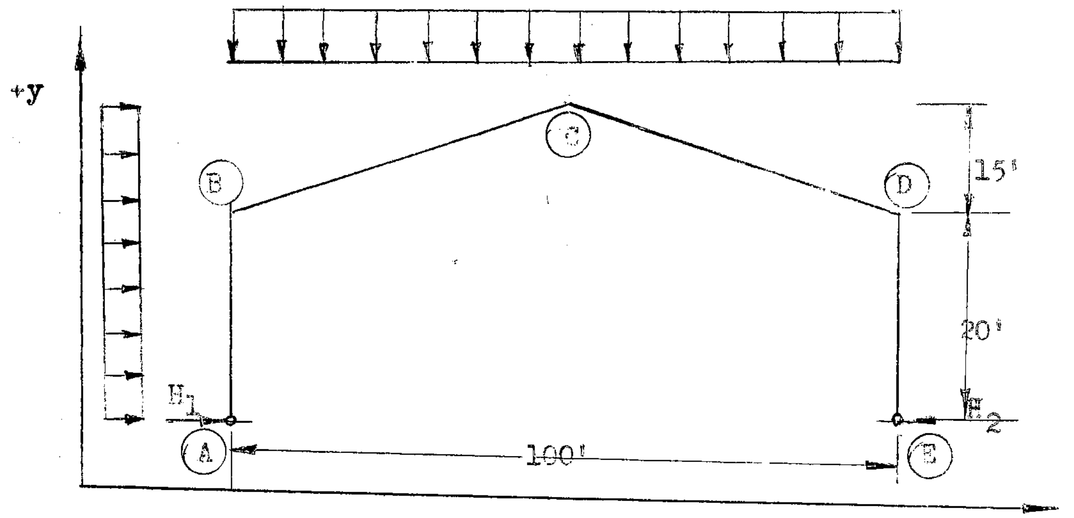


Figure 5-4. Basic Structure

a. Given Data

Vertical load: 1k/ft

Horizontal Load: .6k/ft

TABLE 5-2

DIMENSIONS AND SHAPE FACTORS

Member	Shape	Length	Point	Depth	I(ft^4)	Shape Fact.
AB"	33WF 130	14'	A	33.1"	.323	3
			B"	33.1"	.323	
B"B	$10\frac{1}{2}$x7/8	5.4'	B"	33.1"	.323	2.1
			B	42.0"	.525	
BB'	$10\frac{1}{2}$x7/8	10.2'	B	42.0"	.507	2.4
			B'	24.29"	.1294	
B'C	24WF 94	41.76'	B'	24.29"	.1294	3.00
			C	24.29"	.1294	

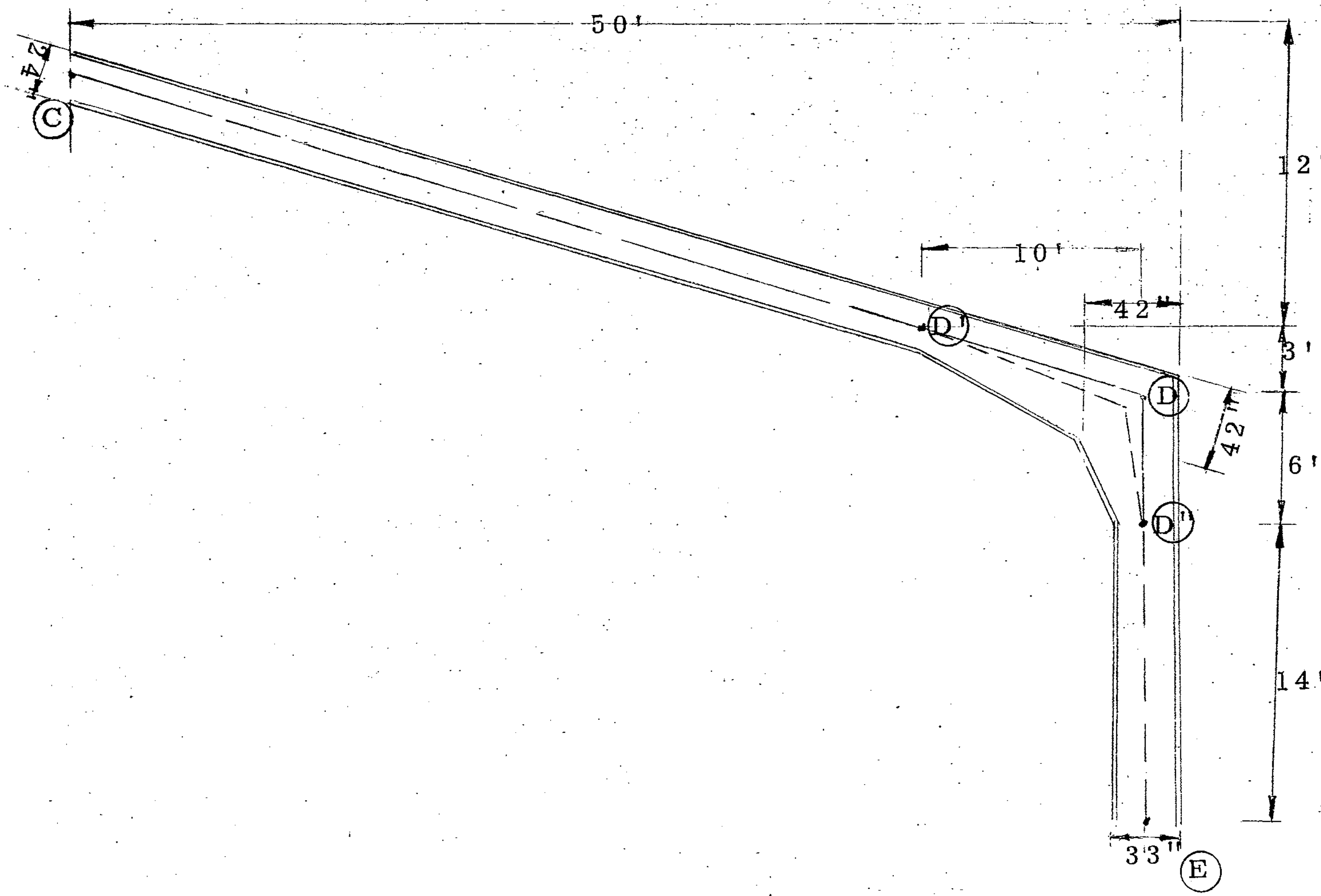


Figure 5-5 Details of structure

(b) Elastic Constants and Load Functions

TABLE 5-3a

ELASTIC CONSTANTS AND LOAD FUNCTIONS

Member	ρ	Table	δ	Flex. Coeff.		C.O.V. Coef.(9)	Ang. L. Coef.	
				f_1	f_2		t_3	t_4
AB"	3.0		1.0	.333	.333	.167	.0416	.0416
B"B	2.1	A-1 A-2	1.26	.2954	.2337	.1312	.0342	.0314
BB'	2.4	A-7 A-8	1.75	.2330	.1190	.0826	.0234	.0179
B'C	3.0		1.0	.3330	.3330	.167	.0416	.0416

TABLE 5-3b

FINAL ELASTIC CONSTANTS

Member	I_o^* / I_o	$F = fL/EI_o$		GEI_o	Ang. Ld. Function	
		$F_{ij}EI_o$	$F_{ji}EI_o$		$\tau_{ij}EI_o$	$\tau_{ji}EI_o$
AB"	2.5	1.86	1.86	.93	27.4	27.4
B"B	2.5	.64	.51	.283	2.16	1.98
BB'	1.0	2.38	1.22	.84	24.1 4.36	18.4 3.32
B'C	1.0	13.90	13.90	6.95	2,780 500	2,780 500

I_o^* Minimum Moment of Inertia of Member

I_o Minimum Moment of Inertia of Structure

(c) Joint-Moment

Vertical reactions at A: 46.32 kips
Vertical reactions at B: 53.68 kips
$M_{B"} = 14H_1 + 58.8$

$M_B = 19.4H_1 + 115$

$M_{B'} = 23H_1 - 254$

$M_C = 35H_1 - 699$

$M_{D'} = 23H_1 - 3.8$

$M_D = 19.4H_1 + 407.1$

$M_{D''} = 14H_1 + 224$

(d) <u>Joint Elastic Weights</u>

$$\bar{P}_{B''} = \bar{P}_{B''A} + \bar{P}_{B''B}$$

$$= 1.86(14H_1 + 58.8) + .64(14H_1 + 58.8) + .283(19.4H_1 + 115)$$

$$+ 2.76 + 2.16$$

$$= 40.48H_1 + 209.2$$

$$\bar{P}_B = \bar{P}_{BB''} + \bar{P}_{BB'}$$

$$= .283(14H_1 + 58.8) + .51(19.4H_1 + 115) + 1.22(19.4H_1 + 115)$$

$$+ .84(23H_1 - 254) + 1.98 + 1.84$$

$$= 56.86H_1 + 18$$

$$\bar{P}_{B'} = \bar{P}_{B'B} + \bar{P}_{B'C}$$

$$= .84(19.4H_1 + 115) + 2.30(23H_1 - 254) + 13.9(23H_1 - 254)$$

$$+ 6.95(35H_1 - 699) + 2{,}780 + 500 + 28.5$$

$$= 633.3H_1 - 5{,}707$$

$$\bar{P}_C = \bar{P}_{CB} + \bar{P}_{CD}$$

$$= 6.95(23H_1 - 254) + 13.9(35H_1 - 699) + 13.9(35H_1 - 699)$$

$$+ 6.95(23H_1 - 3.8)$$

$$= 1{,}294H_1 - 20{,}241$$

$$\bar{\bar{P}}_{D'} = \bar{P}_{D'C} + \bar{P}_{D'D''}$$

$$= 13.9(23H_1 - 3.8) + 6.95(35H_1 - 699) + 2.38(23H_1 - 3.8)$$

$$= .84(19.4H_1 + 407.1) + 2{,}780 + 18.4$$

$$= 633.3H_1 - 1{,}791.3$$

$$\bar{\bar{P}}_{D} = \bar{P}_{DD'} + \bar{P}_{DD''}$$

$$= .84(23H_1 - 3.8) + 1.22(19.4H_1 + 407.1) + .51(19.4 + 407.1)$$

$$+ .283(14H_1 + 224)$$

$$= 56.86H_1 + 769.5$$

$$\bar{\bar{P}}_{D''} = \bar{P}_{D''P} + \bar{\bar{P}}_{D''E}$$

$$= 1.86(14H_1 + 224) + .64(14H_1 + 224) + .283(19.4H_1 + 407.1)$$

$$= 40.48H_1 + 850$$

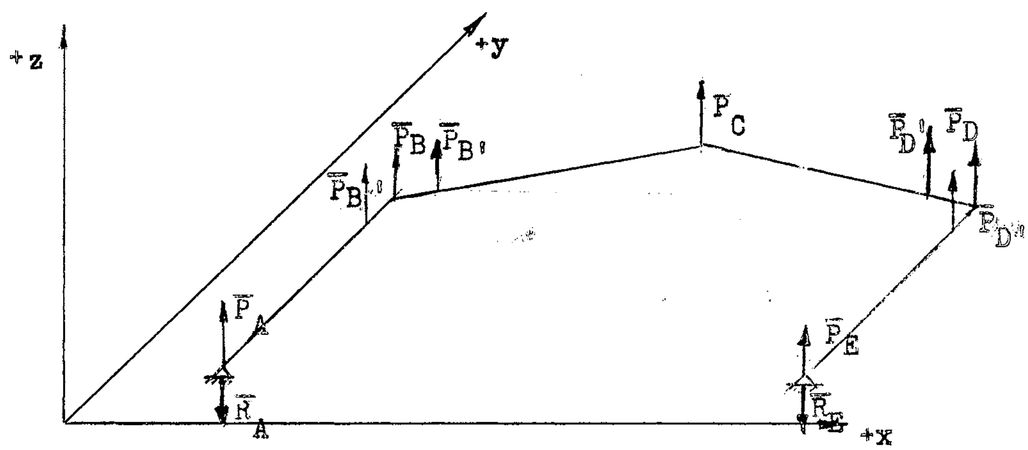


Figure 5-6. Conjugate Structure

(e) <u>Elasto-Static Equation</u>

$$M_{AE} = 0;$$

$$14(\bar{\bar{P}}_{B''} + \bar{\bar{P}}_{D''}) + 19.4(\bar{\bar{P}}_D + \bar{\bar{P}}_B) + 23(\bar{\bar{P}}_{D'} + \bar{\bar{P}}_{B'}) + 35\bar{P}_c = 0$$

$$77{,}817H_1 - 850{,}525 = 0$$

$$H_1 = 10.92 \text{ kips}$$

$$H_2 = 31.92 \text{ kips}$$

CHAPTER VI

SUMMARY AND CONCLUSIONS

The application of the String Polygon Method to the analysis of continuous wedged frames with various end conditions is developed in this thesis. The points of major significance found in this study may be summarized as follows:

1. Joint elastic weights for the wedged member in terms of four moments are written.

2. The interpretation of various end conditions of the real frame in terms of the end conditions of the conjugate frame are developed.

3. The reaction at a given support of the conjugate frame represents the end slope of the real frame at that support about the direction of the reaction. The moment of conjugate frame about a given direction represents the linear displacement of the real frame about that direction.

4. Elasto-static equations and force matrices for the continuous wedged frame with the base fixed are presented.

5. Elasto-static equations and force matrices for the continuous wedged frame with the base hinged are presented.

6. The tables of beam constants for tapered beams of I-section, box section or T-section that are included in this thesis greatly shorten the task of computing elastic weights. These constants can also be used

for the Moment Distribution Method or Slope Deflection Method after a proper transformation.

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